Endogenous Employment and Incomplete Markets

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Abstract

This paper explores the role of effort and human capital as mechanisms to alleviate the idiosyncratic risk faced by individuals in the presence of incomplete markets, and its consequences for the wealth distribution. I construct a DSGE model where effort and human capital determine the probability of being employed the next period. While effort is a flow variable that has to be exerted every period, human capital is a stock variable chosen when the agent is born. I first show how effort and assets are inverse related, and that only rich enough households invest in education. In the stationary equilibrium individuals diversify between market and non-market mechanisms to reduce risk, thus most of the individuals do not need to rely on precautionary savings to smooth consumption. A calibrated version of the model to the US economy approximates the stationary wealth distribution to the empirical one much better than previous studies. The results shed light on the potential implication of combining policies of unemployment insurance and subsidies to education to improve the wealth distribution.

Keywords: Employment, Incomplete markets, Heterogeneity, Endogenous Markov chains

JEL codes: D91, E21, E24, E25, J22

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1 Introduction

Education has been related to higher lifetime earnings through both higher wages and less incidence and duration of unemployment.\(^1\) Since college education is usually obtained by richer households, education creates a stronger tension towards a more unequal distribution of wealth. On the other hand, recent empirical papers have provided evidence on the negative effect that wealth has on the probability of employment once a set of control variables, including human capital, are used (Algan, Chéron, Hairault, and Langot 2003; Bloemen 2002; Bloemen and Stancanelli 2001; Stancanelli 1999). On the theoretical side, such association was rationalized by Lentz and Tranæs (2005) in a search model with savings where effort must be exerted over time to increase the chance of being employed.

The above observations suggest a rather complex relationship between assets and the probability of employment, which seems to be negative in the short-run but positive in the long run. The purpose of this paper is to build a model to explore the joint role of effort and human capital investment as non-market mechanisms used by individuals to deal with their idiosyncratic risk. The analysis provides potential welfare implications for combining public policies related to unemployment insurance and subsidies to education to improve the wealth distribution and the long-run unemployment.

I develop a dynamic stochastic general equilibrium with heterogeneous agents that builds on the framework proposed by Huggett (1993) and Aiyagari (1994) where a riskless asset is used to smooth consumption. However, we depart from the previous models by endogeneizing the transition probabilities between employment and unemployment. In particular we allow the effort and human capital to determine the transition dynamics and thus they can be used by agents as non-market mechanisms to smooth consumption.

\(^1\)According to the Bureau of Labor Statistics, the unemployment rate in 2011 for college graduates was 4.4% versus 9.6% for non-college graduates. On the other hand, the median duration of spells of unemployment was 2.6 months for less than high school graduate, 2.4 for a high school graduate and 1.9 months for individuals with at least some college.
Effort is modeled as a flow variable that has to be chosen every period to maintain a positive probability of being employed, thus following the literature on unemployment insurance (see for example Hopenhayn and Nicolini (1997) and Wang and Williamson (1996)). This can be seen as search effort when the individual is unemployed, or effort on the job when the agent is employed. We assume the level of effort required in the latter case is more effective that the one when the agent is unemployed. This assumption matches with empirical data that has been studied in search models and emphasize the role of the depreciation of human capital during unemployment (Addison and Portugal (1989); Neal (1995)).

On the other hand, human capital is a binary stock variable that can be acquired when the individual is born. It is assumed that it improves the efficiency of effort when looking for a job or maintaining it. Alternatively, a college degree can be interpreted as a variable that generates better shocks. Although human capital has been usually studied as a mechanism to increase earnings, previous empirical work has also pointed out the effect of human capital on employment transitions. For example, Card and Sullivan (1988) estimate the effect of training on the probability of employment for the 1976 cohort of adult male participants in the Comprehensive Employment and Training Act (CETA). They found that the effect is positive, even for people who is already employed. Gritz (1993) also found that participation of women in private training programs increases both the frequency and duration of employment spells.

Although there exists some selection on the individuals that attend to college due to differences in abilities for example, I abstract from this issue. I assume agents are homogeneous in this dimension and I normalize assets by income in order to remove the effect of ability and monetary returns to education. After applying such normalization and removing outliers of the data from the Survey of Consumer Finances (SCF), I found that the calibrated version of the model does a good job on replicating the empirical
wealth distribution, both unconditional and conditional on having a college degree or not.

As it is usual in this literature, asset holdings are restricted to be greater than a lower bound to prevent situations where individuals get indebted forever. This lower bound is used to model a financial friction usually found in reality, and is calibrated accordingly. An upper bound arises naturally from the optimal decisions and the fact that the interest rate is lower than the rate implied by the intertemporal discount factor. This discourages individuals from accumulate forever their asset holdings.

The role of the asset holdings in our model is similar to the one played in previous literature. When the individual is employed she accumulates assets, while she decreases her holdings when unemployed. Therefore, it keeps track of the employment history the individuals have had. However, assets also have a bequest motive in this model. Individuals die with an exogenous probability and newborns inherit the previous wealth. Given the assumptions, we show that only rich enough newborns will invest in education since their marginal value for assets is lower than the one for poor individuals. This generates pressure towards more inequality.

On the other hand, effort has an inverse relationship with assets. If an individual becomes unemployed and has sufficient savings, she will not exert too much effort to find a job and instead use the savings to smooth consumption. However, the ability of the assets to smooth consumption loses importance when they are close to the debt limit. At that point effort plays a major role by increasing the likelihood of being employed next period.

In the stationary distribution most of the individuals will hold a small negative credit balance, while few of them will have positive savings. This means that most of the individuals combine both channels to smooth consumption rather than relying in one of them. As a consequence, the resulting stationary distribution of wealth is much closer to the real one than the wealth distributions obtained by previous studies. Papers that focus
on market mechanisms to alleviate risk usually generate wealth distributions negatively skewed since precautionary savings are the only channel to smooth consumption. On the other hand, empirical papers have shown that only the top deciles have positive savings, while most households hold some degree of debt (Wolff 2010).

1.1 Related Literature

Idiosyncratic shocks and consumption smoothing has been largely studied in the literature. Models of incomplete markets and heterogenous agents have been used to explain the risk premium (Huggett 1993), the benefits of insurance (Hansen and İmrohoroğlu 1992), optimal fiscal policy (Heathcote 2005), and the distribution of income (Aiyagari 1994, Heckman, Lochner, and Taber 1998, Krusell and Smith 1998), among others. The common characteristic of these models is that they use mechanisms affecting the budget constraint to smooth consumption. These mechanisms are usually identified with assets holdings (or credit balances), capital, or savings. However, the labor transitions are always specified exogenously.

Besides the theoretical contribution made by Lentz and Tranæs (2005) on endogenous transitions, other calibrated models of search with savings include Acemoglu and Shimer (2000), Rendon (2006) and Gomes, Greenwood, and Rebelo (2001). However, the do not focus on the wealth distribution and do not include human capital as another source to smooth consumption. Among the papers who have studied the role of human capital on the wealth distribution in the presence of imperfect credit markets we can find Galor and Zeira (1993) and Krebs (2003). However, their main purpose is to explain different growth paths and thus provide an answer to why there exist persistent differences in per capita output across countries. To the best of our knowledge, the relationship between human capital, effort and imperfect credit markets, along with their consequences on the wealth distribution, has not been explored.

The organization of the paper is as follows. The next section describes the
model and the third section defines the equilibrium in this scenario. I then
describe the performed numerical exercise, while section 5 devotes attention
to its computation. In section 6 we show the results and its implications.
The last section concludes.

2 Model

Consider an exchange economy with a continuum of agents with total mass
equal to one who face idiosyncratic risk. There are two commodities: one
perishable consumption good $c$ and asset holdings $a$. Each agent receives an
stochastic endowment $s_t$ at the beginning of each period. Assume the en-
dowment can take two possible values $s_L < s_H$, which are usually associated
with unemployed/employed status, respectively.

Effort $e > 0$ is made in order to increase the probability of having a
good endowment (state) next period. The probability of being employed
next period also depends on whether the agent has a college degree or not,
$h_H$ or $h_L$, respectively. The probability in period $t$ is defined as
$Pr(s_{t+1} = s_H|s_t, h) = P(e_t; s_t, h)$, which is an increasing concave function of the ef-
fort with $P(0; s, h) = 0$ and $\lim_{e \to \infty} P(e; s, h) = 1$. According to the em-
pirical literature, assume that effort to remain employed is more effective
than the effort to become employed when previously unemployed, and effort
is also more effective when the individual has a college degree. Formally,
$P(e_t; s_H, h) > P(e_t; s_L, h)$ for all $h$, $P(e_t; s, h_H) > P(e_t; s, h_L)$ for all $s$. Fi-
nally let the probability be supermodular in $e$, $s$, and $h$.

Individuals discount future at rate $\beta$ and survive next period with proba-
bility $\delta$. When an individual dies it is replaced by an unemployed newborn.
The newly born agent inherits previous wealth and decides whether to obtain
a college degree or not at a cost $\phi$. Agents are altruistic and maximize lifetime
utility of the household. Each individual derives instantaneous utility from
consumption and effort according to an additive separable utility function
$u(c) - e$ that is strictly concave and satisfies Inada conditions. Separability
can be obtained assuming the existence of lotteries and simplifies the analysis
importantly (Lentz and Tranæs 2005). The fact that the disutility of effort is linear is just an innocuous normalization.

Each agent is able to smooth her consumption by holding a single risk-less asset. This asset entitles the individual to receive one unit of future consumption for each unit of asset whose price is \( q > 0 \). The amount of claims held must remain above the limit \( a_{\text{min}} \), a restriction that represent the financial friction faced by individual in addition to the incompleteness of the markets. Therefore, the budget constraint faced by an individual who holds \( a \) claims, has a current endowment \( s \), and chooses consumption \( c \) and future claims \( a' \), is given by

\[
c + qa' \leq s + a
\]  

The agent’s problem can be represented in recursive formulation as

\[
v (a, s, h; q) = \max_{c,e,a'} \{ u (c) - e + \beta \delta [P (e; s, h) v (a', s_H, h; q) \\
+ (1 - P (e; s, h)) v (a', s_L, h; q)] + \beta (1 - \delta) v_0 (a'; q) \}
\]  

subject to (1), \( c \geq 0 \), \( e \geq 0 \), and \( a' \geq a_{\text{min}} \); where

\[
v_0 (a'; q) = \max \{ v (a', s_L, h_L; q) ; v (a' - \phi, s_L, h_H; q) \}
\]

This problem is well defined since \( v (a, s, h; q) \) will inherit the concavity properties of \( u (\cdot) \), while also satisfying discounting and monotonicity (see Stokey, Lucas, and Prescott (1989)). On the other hand, the functional \( v_0 \) can be replaced without loss of generalization by its least concave function. Therefore, the first order conditions are necessary and sufficient, and the
optimal decision rules $c(a, s, h; q)$, $e(a, s, h; q)$, and $a'(a, s, h; q)$ are given by

$$1 \geq \beta P_e(e; s, h) \left(v(a', s_H, h; q) - v(a', s_L, h; q)\right),$$

with equality if $e > 0$

$$u_c(c) q \geq \beta \delta \mathbb{E}[u_c'(c'|e, s, h)] + \beta (1 - \delta) \frac{\partial v_0(a'; q)}{\partial a'},$$

with equality if $a' > a_{\text{min}}$

$$c + qa' \leq s + a$$

The first condition shows the tradeoff between the marginal disutility and the expected marginal benefits of exerting an effort. This condition is similar to the one obtained in the optimal unemployment insurance literature. Using the separability of the utility function we can prove the following lemma:

**Lemma 1** Effort $e(a, s, h; q)$ is decreasing in current assets $a$, whereas households will invest in education if and only if $a \geq \hat{a}$, where $\hat{a}$ is uniquely determined.

The intuition behind this result relies on the fact that the difference on the value function for employed and unemployed people is decreasing in assets, formally $v(a, s_H, h; q) - v(a, s_L, h; q)$ is decreasing in $a$ by supermodularity. In other words, it is less important for rich households whether they are employed or unemployed since they can use their assets to smooth consumption. Therefore the role of effort becomes less important. On the other hand, poor households cannot incur in more debt when they are close to the debt limit. Hence their current state generates great differences in their maximized utility so effort becomes crucial in increasing the probability of being employed. This in turn implies that the probabilities of employment, ceteris paribus, is decreasing in assets.

In contrast, the difference $v(a', s, h_{H}; q) - v(a', s, h_{L}; q)$ is increasing in assets also by supermodularity. Thus richer households will value more education and will be more willing to invest on it.

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The second first order condition is very familiar to the literature that uses asset markets. The limiting behavior of consumption can be characterized by applying the theory of martingales. Let $Z_t = \left( \frac{\beta \delta}{q} \right)^t u_c(c_t) \geq 0$. Therefore, $E_t[Z_{t+1} - Z_t | I_t] = \left( \frac{\beta \delta}{q} \right)^t E_t \left[ \frac{\beta \delta}{q} u_c(c_{t+1}) - u_c(c_t) | I_t \right] < 0$, where $I_t$ is the information set at time $t$, including $c_t$. The previous expectation implies that $Z_t$ is a supermartingale. Since $Z_t$ is nonnegative, we can apply the supermartingale convergence theorem. This theorem states that $Z_t$ must converge almost surely to a nonnegative random variable (Williams, 1991); which leads to the following lemma:

**Lemma 2** In equilibrium $\beta \delta < q$

If $\beta \delta > q$ then $Z_t$ must converge to zero to avoid its divergence. But then this implies that $c_t$ must diverge to infinity. This is obtained by letting the asset holdings go to infinity since the incentives to save are greater than the ones to get more debt. This explosive solution can not be an equilibrium. A similar behavior is obtained if $\beta \delta = q$, see Chamberlain and Wilson (2000) for an exposition.

On the other hand, if $\beta \delta < q$, then $Z_t$ will converge to a nondegenerate nonnegative random variable. This implies that consumption and asset holdings will remain finite, a necessary condition to achieve an equilibrium. In fact, there will be an endogenous upper bound such that no agent would like to save more than such bound (see Ljungqvist and Sargent (2004)). The first order condition also implies that agents will save when facing a good shock and spend savings when facing an adverse shock.

It is important to note that optimal decision rules will depend on their state vector $(a, s, h)$ and on the price of claims $q$. This price will be determined in equilibrium according to a market clearing condition for the asset holdings. The existence of such equilibrium is easy to obtain given the standard properties of the model; however, the equilibrium will not be unique.
Since we are interested in the long run interaction in this economy, we focus only on the stationary equilibrium that we describe in the next section.

### 3 Stationary Equilibrium

The equilibrium in an exchange economy is usually defined as policy rules and prices that clear the markets given some aggregate states. However, the market clearing condition is always changing in this dynamic economy given that the distribution of individuals is always moving. Therefore, a definition of a stationary equilibrium is more appropriate in this context. In this definition we focus on market clearing when the distribution of wealth $\lambda$ is invariant and plays the role of the aggregate variable that depends on the price $q$.

The law of motion of this state vector distribution when $h_{t+1} = h_H$ is described by

$$
\lambda_{t+1} (a', s', h_H; q) = \Pr (a_{t+1} = a', s_{t+1} = s', h_{t+1} = h_H) = \delta \sum_{i=L,H} I (a' = a', s_i, h_H; q) \lambda_t (a, s_i, h_H; q) \Pr (s_{t+1} = s'| s_i, h_H) + (1 - \delta) I (s' = s_L) \sum_{i=L,H} \sum_{j=L,H} \lambda_t (a' + \phi, s_i, h_j; q) I (a' + \phi \geq \hat{a})
$$

where $I (\cdot)$ is an indicator function that takes value 1 whenever the expression inside is satisfied, and 0 otherwise.

A stationary distribution is thus defined as a distribution $\lambda (a, s, h; q)$ such that $T\lambda (a, s, h; q) = \lambda (a, s, h; q)$. The existence and uniqueness of the invariant distribution is established using the approach suggested by Hopenhayn and Prescott (1992) and is obtained because the transition mapping is monotone both in $e$, $h$ and $a$. Therefore, starting from any initial distribution, a sufficient number of iterations will converge to the invariant one.

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2It is similarly defined for $h_{t+1} = h_L$, just replace $h_H$ by $h_L$ and change the last indicator function by $I (a' + \phi < \hat{a})$. 

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Moreover, since $a'(a, s, h; q)$ is bounded, the sequence of averaged assets will also converge.

**Definition 3** A stationary equilibrium is defined by policy rules $c(a, s, h; q)$, $e(a, s, h; q)$, and $a'(a, s, h; q)$; a value function $v(a, s, h; q)$; a price $q$; and a stationary distribution $\lambda(a, s, h; q)$, such that

- The policy and value functions solve the agent’s problem (2)
- Markets clear: $\int_a \sum_{i=1,2} \sum_{j=1,2} a'(a, s_i, h_j; q) \lambda(a, s_i, h_j; q) \, da = 0$
- The stationary distribution $\lambda(a, s, h; q)$ is induced by the policy functions and the endogenous Markov chains generated by $P(e(a, s, h; q); s, h)$.

The first condition states the optimality of the decisions. The second one defines market clearing for assets, which means that the average holdings in the population must be zero. By Walras Law, if the market of loans is cleared, then the market of the consumption good is also cleared by making average consumption equal to the average endowment. The third condition requires that the distribution of assets remains the same over time. For that we need them to remain finite, this is assured by the lower bound and the fact that $\beta \delta < q$. It also plays an important role that $P(e; s_H, h) > P(e; s_L, h)$.

### 4 Numerical Exercise

We calibrate the model according to the previous literature on heterogenous agents, mainly [Huggett (1993)](https://example.com), and unemployment insurance ([Hopenhayn and Nicolini](https://example.com) 1997). We first assume the utility function takes the form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

This is the standard utility function used in this type of problems. According to [Mehra and Prescott](https://example.com) (1985), estimates of the risk aversion coefficient $\sigma$ are around 1.5. The rest of the parameters are calculated according
to periods of 8.5 weeks approximately, that is 6 periods per year. Huggett (1993) chose this length to match the average duration of unemployment spells of 17 weeks (Bureau of Labor Statistics), which is an underestimation of the current average duration of 21.6, but it fits the 5 year trend. For this the endowments were calibrated to \( s_H = 1 \) and \( s_L = 0.1 \), where the last number assumes that individual has access to social programs when he is unemployed. Finally \( \beta = 0.99322 \) to match an annual discount rate of 0.96, and \( \delta = 0.995 \) to match the average death rate.

Hugget also specified an exogenous Markov process where \( \Pr (s_{t+1} = s_H | s_t = s_H) = 0.925 \) and \( \Pr (s_{t+1} = s_H | s_t = s_L) = 0.5 \). This calibration replicates a coefficient of variation for the annual earnings of 20%, which is close enough to the actual data. It also generates an annual average endowment of 5.3; therefore, we set \( a_{\text{min}} = -5 \) to simulate the financial friction. This bound is close in equilibrium to the natural borrowing limit of \(-\frac{s_L}{r}\) described by Aiyagari (1994).

In order to obtain similar quantitative results, we calibrate our endogenous Markov chain to find similar probabilities. We model the probability of having a high state tomorrow as a cdf of an exponential distribution with parameter \( \mu (s_i, h_j) = s_i h_j \), that is \( P (e; s_i, h_j) = 1 - \exp^{-s_i h_j e} \), where \( h_L = 10 \) and \( h_H = 12.5 \). This parameterization satisfies our initial assumptions of first order stochastic dominance and the ones described by Hopenhayn and Nicolini (1997) to characterize the optimal unemployment insurance, it also targets the percentage of people that acquire college education. The calibration suggests that, absent of monetary returns to education, having a college degree has a 25% return on finding a job. Moreover, as shown in the next section, the optimal probabilities in equilibrium will wander around Huggett’s calibration. Finally, the cost of education is set to \( \phi = 4 \) to match the average cost of public college relative to average income (see the 2011 report from the College Board). Note this number allows individuals with some degree of debt to invest in education.
5 Computation

To find the optimal policy rules we first set a candidate for \( q \), say \( q_1 \), belonging to a plausible interval of equilibrium prices. We then use value function iteration to obtain the optimal policy rules. Since all the desired properties of the value function are satisfied, convergence is achieved independently of the initial guess for the value function. To compute the solution we discretize the choice of \( a \), obtain \( c \) from its first order condition and consumption from the budget constraint. The grid must be fine enough to achieve smooth policy functions.

As pointed out before, there exists a natural upper bound for \( a \). Optimal future assets for an employed agent start above the 45° line (when current assets are negative), and then crosses this line for some positive level of current holdings, say \( a_{\text{max}} \). On the other hand, an unemployed agent will always reduce her holdings to maintain her consumption. See Fig. 2 in the appendix for an example of an optimal policy rule for asset holdings.

This shape of the optimal policy implies that \( a_{\text{max}} \) plays the role of a fixed point when an agent is always employed. Moreover, it also plays the role of an upper bound since once the agent receives a bad shock she will decrease her assets. Hence, an agent with any initial wealth will converge to the interval \([a_{\text{min}}, a_{\text{max}}]\), and remain there forever. This upper bound can only be computed by experimentation and thus the upper bound of the grid is set large enough to include the fixed point.

After obtaining the optimal decision rules we compute the stationary distribution. To obtain it we simulate an economy of 100000 agents and iterate for 200 periods. The initial distribution of states and assets will not matter for the convergence. We first fix a set of two i.i.d shocks \( \epsilon_{i,t} \) and \( z_{i,t} \) with a uniform distribution between 0 and 1 for each individual and each period. We then interpolate the optimal decision using the optimal policy rules and the current asset holdings and state. If \( z_{i,t} > \delta \) the individual

\footnote{We also chose a longer horizon without obtaining significant differences.}
dies and a newborn must decide whether to acquire the college degree or not according to the inherited wealth. If this is not the case then I proceed to compare the first i.i.d shock with the probability associated with the optimal effort and the current state. If $P(e(a_{i,t}, s_{i,t}, h; q); s_{i,t}, h) \leq \epsilon_{i,t}$ then $s_{i,t+1} = s_L$, otherwise the agent will be employed.

After the stationary distribution is computed we calculate the excess demand for assets given the initial price $q_1$. Then we follow Huggett’s process of bisection: if the excess demand is positive we increase the price $q$, if it is negative we decrease it. This algorithm follows the conjecture that the excess demand of assets is negatively correlated with its price. Although this is hard to prove in general, this is the case in the interval we examined, and it has been also true in related papers that follow the same methodology (see for example [Huggett (1993) and Aiyagari (1994)]). The process continues until excess demand is approximately 0 and the difference of the updated price is less than 0.001.

6 Results

Fig. 1 shows the concavity of the value function that permits the contraction to find the fixed point. It also shows how utilities diverge when asset holdings are close to the lower limit, a result that is intuitive after examining the policy rules. The optimal asset policy is shown in Fig. 2 and it follows the behavior described in the previous section. It shows how individuals with low states will decrease their holdings until the lower limit, while individuals with good shocks accumulate holdings until they reach the upper bound. This is a characteristic of the models in this branch of the literature.

In our model we also explore a different non-monetary mechanism used by individuals to alleviate risk. Individuals use effort to increase their probability of being employed next period, especially when their level of assets is approaching its lower limit. The optimal probability of employment conditional on human capital is decreasing on the asset holdings and is lower for unemployed individuals since, by assumption, is harder to change their
status. These probabilities are shown in Fig. 3 and wander around the probabilities calibrated by Huggett (1993), providing a good approximation of the steady state. They also show how the individual increases them when asset holdings are close to the lower bound.

As a consequence of this optimal strategy for risk bearing, consumption has very little variation across different types of individuals, except for unemployed agents whose asset holdings are close to the lower limit. Fig. 4 depicts the optimal consumption. Fig. 5 shows the excess demand of holdings, which depends negatively on the price. The price of assets that clears the market is 0.9933, which is equivalent to an annual interest rate of 4.1%. This rate is higher than the one obtained by Huggett (1993) since individuals have more incentive to acquire debt instead of save given the other existing mechanisms to smooth consumption.

The obtained percentage of individuals with a college degree is 30.6% which is close enough to the real one (29%). The simulated rate of unemployment is 5.5%, consistent with unemployment rates for developed economies. The model generates an unemployment rate of 4.6% and 6% for college and non-graduates, respectively. The generated gap between these two rates is not enough, since these numbers are currently 4.9% and 9.5% for the US. The model is also robust to small perturbations in the parameters. In an alternative scenario where \( h_L = 16 \) and \( h_H = 20 \), the rate of unemployment decreases to 5%, which is decomposed on 4.5% and 5.7% for college graduates and non-graduates, respectively.

The distribution of wealth in the stationary distribution differs from the one found by Huggett (1993) and the one potentially generated by the class of models where consumption can only be smoothed through market mechanisms. These models generate distributions skewed to the left since they must accumulate precautionary savings to deal with their idiosyncratic shocks. In

\[4\text{Current unemployment for US is 7.6%, which is higher than the trend observed in previous years.}\]
contrast, when the transitions are endogenous, individuals will diversify between the market and the non-market mechanisms.

Fig. 6 shows how the wealth distribution in our model is skewed to the right, approximating better the real wealth distribution shown in Fig. 9. The empirical wealth distribution is calculated according to the SCF (2010) after removing outliers using the interquartile range method and normalizing household wealth by income, and thus be able to compare the model with the data. The results are robust if we calculate the household wealth following Wolff (2010) and Sierminska, Michaud, and Rohwedder (2008). Results are also robust if only non-home wealth is included.

This suggests that in the long run individuals are not afraid of becoming indebted since they have an extra mechanism to smooth consumption. At the end, the incomplete markets partial insurance is successfully complemented by the effort and human capital. This result is a consequence of the convexity properties of the sets. In our model it can be traced to the concavity of the probability transition to the employed state, as well as the concavity of the utility function.

Fig. 8 shows the disaggregation of the wealth distribution for college graduates and non-graduates. It is found that distribution for the former is more dispersed and less skewed. Thus having college education seems to increase the expected wealth but creates more uncertainty at the same time. Remarkably, this distribution follows closely the empirical one (Fig. ??), even though the calibration was not targeting any moment of the (conditional) wealth distribution.

7 Concluding remarks

I have studied a model of heterogenous agents who face idiosyncratic risk and smooth their consumption using a riskless assets and non-market mechanisms. Effort is a flow variable that must be exerted every period to obtain or maintain a job, whereas human capital increases the efficiency of effort in obtaining a job and persists until the individual dies. We found that effort
and assets have an inverse relationship and it is shown how the investment in a college degree depends on its cost. If the cost is sufficiently high, as it appears to be according to the calibration, then only rich-born agents acquire education.

In the stationary equilibrium agents diversify among these mechanisms and as a result I obtain a distribution of wealth that is not as skewed as the one generated by previous models. In particular, the median individual holds a small negative credit balance and exert a medium amount of effort. This result contrasts with the ones previously obtained where the median individual holds a positive credit balance. Therefore, our framework replicates much better the real distribution of wealth.

The analysis also shows how the distribution of wealth for college graduates is more dispersed and less skewed than the distribution for non-graduates. The model could be used as a benchmark to evaluate the combination of unemployment insurance policies with subsidies for education to improve the wealth distribution. It first suggests how asset holdings could be used as a proxy to unobservable effort, and how education can be used as long-run insurance.

References


A Figures

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Figure 8: Conditional stationary distribution of assets for the calibrated model
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