Conciliating Absolute and Relative Poverty: Income Poverty Measurement with Two Poverty Lines

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Main goal

- The paper proposes a new poverty index simultaneously based on two poverty lines: the absolute line and the relative line.
- The idea on which the index is derived is that the poverty contribution of an absolutely poor individual should be **larger** than the poverty contribution of an individual relatively poor, regardless of the income standard in their respective societies.
- Mainstream poverty indices violate this property (they are not hierarchical), yielding counter-intuitive comparisons: **the (relative) poverty rate in Brazil is higher than in Ivory Coast**.

The different contributions to the poverty index I

- Given an income/consumption distribution y, the proposed index, an extended Head Ratio poverty index, is equal to the fraction of absolutely poor individuals HC^{abs} plus the fraction of individuals who are **only** relatively poor HC^{rel} multiplied by an endogenous weight w:

$$HC^{ext} = wHC^{abs} + (1 - w)HC^{rel}$$

- The weight $w(y)$ is a linear function of the distance between the average income of those only relatively poor the absolute poverty line and the relative line.
- Considering a general poverty index

$$P(y) = \frac{1}{n} \sum_{i=1}^{n} P(y_i, \bar{y})$$

The different contributions to the poverty index II

- $P(y_i, \bar{y})$ represents the contribution of individual i to the poverty index, $\bar{y}$ the median income of the country when i resides.
- In this framework where **two poverty lines are involved**:

$$P(y_i, \bar{y}) = (1 - u_\lambda(y_i, \bar{y}))^\alpha$$

where

$$u_\lambda = \begin{cases} \lambda \frac{y_i}{z_\alpha} & \text{if } y_i < z_\alpha \text{ (absolutely poor)} \\ \lambda + (1-\lambda) g(y_i, \bar{y}) & \text{if } z_\alpha \leq y_i \leq z_r \text{ (only relatively poor)} \end{cases}$$

where

$$g(y_i, \bar{y}) = \frac{y_i - z_\alpha}{z_r - z_\alpha}$$

and z_α and z_r are the absolute and relative poverty line, respectively.

The different contributions to the poverty index III

- Suppose now $\lambda = 0$ and $\alpha = 0$
- The contribution of individual i to the extended poverty index will be:

$$P(y_i, \bar{y}) = \begin{cases} 1 - \lambda \dfrac{y_i}{z_\alpha} = 1 & \text{if } y_i < z_\alpha \\ 1 - \dfrac{y_i - z_\alpha}{z_r - z\alpha} = \dfrac{z_r - y_i}{z_r - z_\alpha} & \text{if } z_\alpha \leq y_i \leq z_r \end{cases}$$

A trivial example: Brazil versus France I

- Consider now a very simple example in which we compare two countries: say Brazil (B) and France (F);
- The absolute poverty line z_α is fixed at 2$ US international and the two relative poverty lines fixed at $z_r^B = 10\$$ and $z_r^F = 30\$$
- **Case a)**: we now compare two **absolutely poor individuals**, a Brazilian and a French, who both have income equal to 1$: $y_i^B = y_i^F = 1\$$
- Their contribution to the extended poverty index is the same and equal to 1, independently on the country where they live:

$$P(y_i^B = 1\$) = 1 - \lambda\frac{1}{2} = 1, \text{ if } \lambda{=}0$$

$$P(y_i^F = 1\$) = 1 - \lambda\frac{1}{2} = 1, \text{ if } \lambda{=}0$$

A trivial example: Brazil versus France II

- **Case b)**: the two individuals have both 8\$: $y_i^B = y_i^F = 8\$$, that is they are not absolutely poor but **only relatively poor**;
- Their contribution to the poverty index is not equal anymore:

$$P(y_i^B = 8\$, z_r^B = 10\$) = \frac{z_r^B - y_i^B}{z_r^B - z_\alpha} = \frac{10-8}{10-2} = 0.25$$

$$P(y_i^F = 8\$, z_r^F = 30\$) = \frac{z_r^F - y_i^F}{z_r^F - z_\alpha} = \frac{30-8}{30-2} = 0.78$$

- The French citizen has a bigger weight and therefore matters more for the poverty index, since his income is relatively closer to the absolute poverty line (his income is closer to the absolute poverty line than to the relative poverty line) than the income of the poor Brazilian.

A trivial example: Brazil versus France III

- **Case c)**: a French citizen with 20$ contributes to the poverty index with a weight equal to:

$$P(y_i^F = 20\$, z_r^F = 30\$) = \frac{z_r^F - y_i^F}{z_r^F - z_\alpha} = \frac{30 - 20}{30 - 2} = 0.35$$

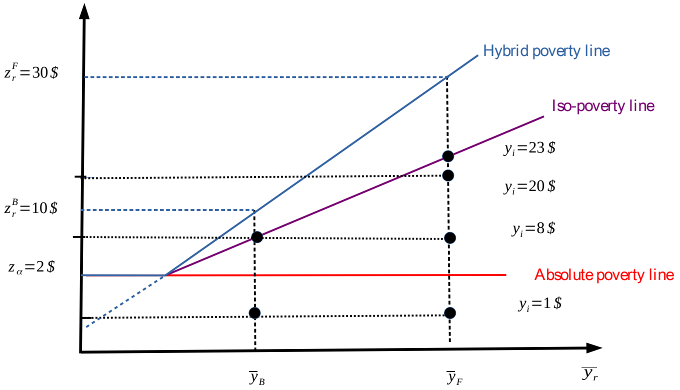
that is less than the previous one since his income is closer to the French relative line $z_r^F = 30$.

- In order to have a weight equal to 0.25 the French poor should have an income equal to 23$:

$$P(y_i^F = 23\$, z_r^F = 30\$) = \frac{30 - 23}{30 - 2} = 0.25$$

This French poor with income equal to 23$ and the Brazilian poor with income equal to 8$ lay on the same **iso-poverty line**.

A trivial example



Estimation of poverty

	BRAZIL	FRANCE
Population	10	10
Relative Poor	2 (8$)	2 (8$)
Absolutely Poor	1	1

- with this hypothetical situation we will have: $HC^A = 10\%$ both in France and in Brazil, $HC^R = 20\%$ both in France and Brazil, while

$$HC_B^{ext} = \frac{1 + 0.25}{10} = 12.5\%; \; HC_F^{ext} = \frac{1 + 0.78}{10} = 17.8\%$$

- This trivial example shows that "controversial situations" (poverty in France is higher than in Brazil) can still happen.

Comments and suggestions I

- Very nice paper: well written and very accurate.
- All the properties of the extended index are properly enhanced and all the proofs are reported in the Appendix.
- The idea of combining the two poverty line and have a single complete measure is interesting and well motivated in the introduction of the paper.
- However, we have seen that controversial situations can still happen when the incomes of the **only relative poor** people are closer to z_α than to the relative poverty line z_r.
- My suggestion is to look also at polarization indexes to have a more detailed view of the entire population and understand how the relative poor in a country are distant from the mean/median of the entire distribution.

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Comments and suggestions II

- A curiosity: which is the gain of the proposed extended index with respect to the Poverty Gap Index

$$PGI = \frac{1}{n}\sum_{j=1}^{q}\left(\frac{z-y_i}{z}\right)$$

(q the number of poor) that also consider the distance from the poverty line