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Abstract

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1 Introduction

Intangibles are an increasingly important component of U.S. economic activity. The Bureau of Economic Analysis (BEA) reports that "intellectual property products" rose from 13% of US non-residential fixed investment in 1980 to 27% in 2000 and to 33% by 2017. These intangible investments may have profound implications for the economy now and for its future growth. They certainly raise challenges for how we measure GDP and productivity growth.

Current national accounting practice appropriately treats intangibles as a durable asset, which is why they are now included as a category of investment rather than as an intermediate input. We argue that current practice inappropriately treats them as a rival factor of production, even though they are typically nonrival. We develop a theoretical model that illustrates this issue and draws out its implications for measuring intangibles and for measuring productivity growth. The model serves as the foundation for an accounting framework that treats intangibles as nonrival assets.

2 Summary Argument

What do we mean by intangibles? While they are often lumped together with services, Hill (1999) makes the case for distinguishing them from services and from tangible goods. In his words:

> They are the originals created by authors, composers, scientists, architects, engineers, designers, software writers, film studios, orchestras, and so on. These originals are intangibles that have no physical dimensions or spatial co-ordinants of their own and have to be recorded and stored on physical media such as paper, films, tapes or disks. They can be transmitted electronically. (pg. 427)

The two fundamental properties of intangibles are durability and nonrivalry. Intangibles are even more durable than physical capital since they are not worn out by repeated use over time. The same piece of software can be run forever, although if it becomes obsolete due to the creation of better software a user may choose not to run it. Intangibles are nonrival since they can serve multiple users at once. The same piece of software adopted by one firm can be transmitted to others with no additional coding required. A firm's use of the software is not affected from use by others, although a firm may earn more if its competitor doesn't have access to the software.

The first of these fundamental properties is now reflected in the treatment of intangibles in the national income and product accounts (NIPA). Traditionally considered as intermediate consumption, twenty years ago the BEA began to count spending on intangibles as investment and to capitalize it into stocks of intangible assets. While such spending initially covered only business software, the set of intangibles has been broadened to include research and development (R&D) as well as entertainment, literary, and artistic originals.[1] Corrado, Hulten, and Sichel (2009) illustrate how this shift, which creates an additional factor of production, expands measured GDP.

The second of these fundamental properties is largely ignored in the national accounts and in related measures of productivity.[2] Yet, nonrivalry of intangible technology plays a key role in the Schumpeterian new growth literature of Romer (1990), Grossman and Helpman

[1]Business spending on software was reclassified from intermediate consumption to investment starting with the 1999 comprehensive revision of the NIPA. Research and Development and entertainment, literary, and artistic originals were included in investment starting with the 2013 comprehensive revision. Investment spending on intangibles enters the NIPA under the investment heading of intellectual property products. See Department of Commerce (2019) for details.

[2]The words "intangible" and "nonrival" do not appear in the index of the international System of National Accounts 2008 (SNA 2008; Commission of the European Communities et al., 2009), although there are entries for "knowledge" and "intellectual property products." Under the heading of "Originals and copies recognized as distinct products" (pg 587) it recommends "if a copy is sold outright and expected to be used in production for more than a year then it should be treated as a fixed asset." This treatment indicates that SNA 2008 considers intangible goods to be rival.

(1991), and Aghion and Howitt (1992). We follow Nakamura (2010), who calls for such a Schumpeterian paradigm in the treatment of intangibles in the national accounts, and we show how to proceed in this direction by embracing the nonrivalry of intangibles in a tractable dynamic accounting framework. To illustrate the relevance of this approach for national accounting practice, we develop parallel models of the economy, similar except that one treats intangibles as rival and the other treats them as nonrival. While the treatment of intangibles as rival or nonrival is irrelevant for measuring the level of GDP, we show that it has first-order effects on the measurement of multifactor productivity (MFP).

The rival-nonrival distinction also matters for what economic transactions make sense to include in the stock of intangibles. Our analysis argues for a narrower scope than is currently applied by BEA to the NIPA's. In what follows, we illustrate these points with some simple numerical examples, which are essentially static. After that we sketch out their implications for growth accounting.

2.1 Accounting for Intangibles: Illustrative Examples

To be explicit, we format the accounts for each of the examples that follow using an imaginary input-output table (the use of commodities by industries).

2.1.1 Tractors

We fix ideas by reviewing how the national accounts portray a tangible durable good like a tractor. Consider a simple economy consisting of a tractor producer, a potato farmer, and an intermediary who buys the tractor (a durable capital good) and rents it to the farmer. For simplicity, assume that all of these activities occur within the same year. In reality, the intermediary may be merged with the tractor producer (which then buys the tractor from itself and rents it to the farmer) or with the farm (which then buys the tractor and rents it to itself). Including the intermediary facilitates this flexibility of interpretation.

The accounts for this economy are shown in Table 1. The first three rows correspond to the three commodities (tractors, tractor services, and potatoes) while the first three columns correspond to the three industries. Starting with the first, the tractor producer hires labor at a cost of 40 to produce a tractor worth 50. Its gross operating surplus of 10 is some combination profit and return on plant and equipment. The tractor is sold to the tractor services intermediary, whose spending on it is final demand, an investment of 50. The tractor services intermediary generates value added of 30 by leasing the tractor to the farmer, who combines tractor services (intermediate consumption for the farmer since he doesn't own the tractor) and labor to produce potatoes worth 100 for final consumption. Measured as either the sum of value added, the sum of income, or the sum of final output, GDP is 150. Depending on whether we merge the intermediary with the tractor producer, merge it with the farm, or leave it on its own determines which of the three firms makes the investment, but in any case total investment remains 50, the tractor's value.

Table 1: Tractors as Tangible Capital

	Tractor manufacturing	Tractor services	Farming	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Tractors	0	0	0	0	0	50	50	50
Tractor services	0	0	30	30	0	0	0	30
Potatoes	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	0	30					
Compensation of employees	40	0	70	110				
Gross operating surplus	10	30	0	40				
Total value added	50	30	70	150			150	
Total industry output	50	30	100					180

2.1.2 Drug Discovery

Turning from tangibles to intangibles, consider an economy consisting of a drug developer, a drug producer, and a drug retailer. The three commodities are original drug entities, bulk drugs, and drugs packaged for retail. We can imagine drug production merged with drug discovery or with drug retailing, but the accounts are more transparent if we keep this production activity distinct from the other two. Table 2 shows the accounts for this economy when we treat drug discovery (an R&D activity) as intermediate consumption, as would have been its treatment in the NIPA's prior to 2013. Starting with the first column, the drug developer hires labor at a cost of 40 to develop the formula for a new drug worth 50. The drug producer buys the formula (say, protected by a patent) and hires labor at a cost of 10 to produce a quantity of this drug worth 30. This activity yields a gross operating surplus of -30. (Presumably the drug producer expects to use this formula in future years, which is why treating its purchase as a current expense is problematic.) The drug retailer buys the drugs for 30 employs 60 in labor and sells drugs to households for 100, generating value added of 70. GDP of this economy is simply the 100 of final consumption. The value of the original drug entity, the outcome of R&D by the drug developer, is netted out of GDP since the purchase of the formula was treated as intermediate consumption.

Treating the discovery of an original drug entity more like the production of a tractor, as in current BEA procedure, appears more appropriate as we now demonstrate. Table 3 shows the accounts when R&D, or the value of an original formula, counts as investment. The first column remains the same as Table 2, but the second column removes the purchase of the formula from intermediate consumption. It shows up instead as 50 of investment by the drug producer, a final output of the economy.[3] What had been a loss for the drug producer

[3]To highlight the similarity to the tractor example, we measure R&D by the value of the original drug entity it created, which is 50, rather than by the cost of the labor employed, which is 40. In practice, the value of the original may be hard to come by so that the R&D spending of 40 might be used as a proxy.

Table 2: Original Drug Entities as Intermediate Consumption

	Drug developers	Drug producers	Drug retailers	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Original drug entities	0	50	0	50	0	0	0	50
Drugs bulk	0	0	30	30	0	0	0	30
Drugs retail	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	50	30					
Compensation of employees	40	10	60	110				
Gross operating surplus	10	-30	10	-10				
Total value added	50	-20	70	100			100	
Total industry output	50	30	100	180				180

is now a gross operating surplus of 20. Nothing changes in the third column depicting the operations of the drug retailer. GDP rises from 100 to 150 (becoming the same as for the tractor economy) due to treating spending on the drug formula as investment. Note that in this example it was unnecessary to make a distinction between a tractor (a rival good) and an original drug formula (a nonrival good). The key point here was instead the conceptual advantage of treating an original drug formula as a durable good, like a tractor.

The rival versus nonrival distinction is clearly present, however. Suppose consumer demand for potatoes and for drugs both doubled. To equip the larger farm we'd need to produce another tractor, but to equip the larger drug factory the same drug formula would do. The distinction thus rears its head when we consider growth, which we will turn to later. But, it also comes up in a more subtle way in these static accounting exercises when we consider an intangible good produced with intangible originals. In our example of drugs, the ultimate product was not itself a formula, but rather a tangible bottle of pills. The example of software, however, brings up a new issue since the ultimate product is no more tangible than the original.

Table 3: Original Drug Entities as Intangible Capital

	Drug developers	Drug producers	Drug retailers	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Original drug entities	0	0	0	0	0	50	50	50
Drugs bulk	0	0	30	30	0	0	0	30
Drugs retail	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	0	30					
Compensation of employees	40	10	60	110				
Gross operating surplus	10	20	10	40				
Total value added	50	30	70	150			150	
Total industry output	50	30	100	180				180

2.1.3 Software

Consider an economy with one firm developing software for accountants and any number of accounting firms providing services to households. The software developer creates an original code much like the drug developer discovers the formula for a new drug entity. We introduce two intermediaries that operate between this developer of software and the accountants that use it. The first intermediary is a software copying firm that purchases the original code and makes copies to sell. Since the software copies are themselves durable (like the tractor), we also introduce a software services firm that purchases the software copies and provides software services to each accounting firm, via a single-year licence. This rather elaborate industrial structure puts the current BEA procedure in the best light.

Table 4 shows the accounts for this economy, with software treated as durable but rival, as in the current NIPA. The software developer employs labor at a cost of 40 to create an original code worth 50. The software copying intermediary buys it, generating final investment spending of 50. This intermediary sells the copies to the software services firm for 50. This spending of 50 by the software services intermediary is also considered investment. The software services intermediary licences the software to the accountants for 30 (15 per accountant if there are two of them).[4] Accounting services generate 100 of final spending on consumption. Adding in final investment spending of 100 (on the original and the copies) GDP is 200.

Table 4: Software as Rival Intangible Capital

	Software developers	Software copiers	Software services	Accounting services	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Software originals	0	0	0	0	0	0	50	50	50
Software copies	0	0	0	0	0	0	50	50	50
Software services	0	0	0	30	30	0	0	0	30
Accounting services	0	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	0	0	30					
Compensation of employees	40	0	0	70	110				
Gross operating surplus	10	50	30	0	90				
Total value added	50	50	30	70	200			200	
Total industry output	50	50	30	100	230				230

At one level the accounting procedure described above is eminently logical. We already saw the advantage of treating R&D as an investment (in the context of drug discovery) and spending on copies of software also has the features of investment spending, yielding services into the future for the purchaser. On the other hand, an original software code is the essence of a nonrival good. Should we really consider simply making copies of it to be an investment activity that expands GDP? To explore this question we consider what happens if only spending on the original code is treated as investment.

Table 5 shows the accounts for that case. The activities of the software developer and software copier remain unchanged from the case where software is treated as a durable rival

[4]Unlike drug production, which required labor of 10 to transform a drug formla into pills for retail, the two intermediaries in the software example require no labor to transform original code into software services. Those numbers reflect the reality that software can typically be downloaded by users directly from the web site of the software developer.

good. But now the purchase of the software copies by the software services intermediary becomes intermediate consumption rather than investment. It sells only single-year licenses to the accounting firm for a total of 30, hence suffering a gross operating surplus of -20. Investment, of course, falls from 100 to 50, while GDP drops to 150, down from 200. An unattractive feature of these accounts is that they portray the software services intermediary as spending on intermediate consumption for software copies that will yield payoffs into the future. That awkward feature is due to our keeping the industry structure unchanged from the structure that was designed to justify current BEA procedure. We turn next to a simpler and more natural accounting structure.

Table 5: Software Originals as Nonrival Intangible Capital

	Software developers	Software copiers	Software services	Accounting services	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Software originals	0	0	0	0	0	0	50	50	50
Software copies	0	0	50	0	50	0	0	0	50
Software services	0	0	0	30	30	0	0	0	30
Accounting services	0	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	0	50	30					
Compensation of employees	40	0	0	70	110				
Gross operating surplus	10	50	-20	0	40				
Total value added	50	50	-20	70	150			150	
Total industry output	50	50	30	100	230				230

Suppose we merge software copiers into software services. Likewise, we will drop software copies as a separate good, thus breaking with SNA 2008 orthodoxy (which treats the original and copies of it as distinct assets). The idea is that in practice software services derive directly from the original code. After all, these days copying activity takes place by the user, who simply purchases the right to download the code to her device. In the models introduced below, we will interpret the value of the services provided by the software as a royalty payment earned by the intangible asset which we think of as the original code. Of course, for many types of software the purchase involves a multi-year agreement. We argue that isn't a fundamental issue of concern to the national accounts. Its like paying up front for a three-year subscription to *The Economist* magazine. That's simply a financial arrangement that minimizes transactions costs, not a real investment.

Table 6 shows the simplified accounts for this case. The activity of the software developer is unchanged, but now the software services intermediary purchases the original, making an investment of 50. That intermediary then sells 30 of software services to the accounting firms, generating value added of 30. GDP remains 150, while total gross output declines as we net out the intermediate transactions that had previously taken place between the two intermediaries.

Table 6: Software Originals as Nonrival Intangible Capital, Simplified

	Software developers	Software services	Accounting services	**Total inter-mediate**	Con-sumption	Invest-ment	**Total final uses (GDP)**	**Total commodity output**
Software originals	0	0	0	0	0	50	50	50
Software services	0	0	30	30	0	0	0	30
Accounting services	0	0	0	0	100	0	100	100
Total Intermediate inputs	0	0	30					
Compensation of employees	40	0	70	110				
Gross operating surplus	10	30	0	40				
Total value added	50	30	70	150			150	
Total industry output	50	30	100	180				180

This simplified table makes our case for treating as investment only the value of the original code, but not purchases of the copies of it. The implication is that intangible investments should include R&D but not software purchases. As of 2017, that means dropping about 20% of what is currently considered investment in intellectual property products.

2.2 Growth Accounting with Intangibles

Having used these simple numerical examples to illustrate the static accounting issues in the treatment of intangibles, we now turn to the implications for growth accounting of treating intangibles as nonrival. Thus, we will always consider the intangible to be the original, not the copy. Here we keep the analysis quite stark, postponing a complete model until the subsequent sections.

2.2.1 The Production Function

Consider a firm that produces a good (for example a retail drug or accounting services) using an intangible (such as a drug formula or software code) to improve its product or to automate its operations. The firm's production function might be represented as:

$$y = F(l, k, x),$$

where y is units of output, l is employment, k is the quantity of the intangible, and x represents all other inputs used by the firm. What can we say about the production function F?

Suppose the firm wishes to double its output. The replication argument says it can do so by doubling l, k, and x, for example by simply setting up a second establishment elsewhere with the same set of inputs as the original. That reasoning suggests that F is homogenous of degree 1 in all inputs:

$$2y = F(2l, 2k, 2x).$$

That's the end of the story if intangibles are treated as rival.

But, what does it mean to double k? Why can't the same drug formula be used to serve twice as many drug retailers? Why can't the original software code be used to serve twice as many accounting firms? If the originals are nonrival they can, hence:

$$2y = F(2l, k, 2x).$$

With nonrival intangibles F is homogenous of degree 1 in l and x alone, as the same intangibles k are capable of operating at any scale. This argument is essentially from Romer (1990), where he expounds on the nonrivalry of technology.

2.2.2 Measuring Productivity Growth

Consider an economy producing goods G and creating intangibles I. The factors of production are labor, employed at wage w_t in both sectors $L_t = L_t^G + L_t^I$ and the stock of intangibles K_t, used only to produce goods. Investment in intangibles $w_t L_t^I$ leads to higher K. Letting Y_t^G denote the value of goods-sector output, the value of GDP is:

$$Y_t = Y_t^G + Y_t^I = Y_t^G + w_t L_t^I.$$

In terms of payments to factors (assuming no pure profit):

$$Y_t = w_t L_t + \rho_t K_t = w_t L_t + \left(Y_t^G - w_t L_t^G\right),$$

where $\rho_t K_t$ are payments to intangibles, equal to the revenue of the goods sector less wage costs.[5] In this setting, treating intangibles as rival or nonrival is irrelevant for measuring GDP, given data on Y_t^G and Y_t^I.

The distinction between rival and nonrival intangibles starts to matter when we turn to the production function. With intangibles rival, the production function for real output of the goods sector would be:

$$y_t^G = A_t F(L_t^G, K_t),$$

where F is homogeneous of degree 1 in the two factors of production. (We don't carry around any additional factors x in the analysis that follows.) Assuming competitive factor and product markets, the growth of MFP is given by Solow's residual:

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{y}_t^G}{y_t^G} - \alpha_t \frac{\dot{L}_t^G}{L_t^G} - (1 - \alpha_t)\frac{\dot{K}_t}{K_t},$$

[5]While in either case $\rho_t K_t$ is the residual revenue of the goods sector, these payments are for rental if intangibles are treated as rival while they are royalties if intangibles are treated as nonrival.

where $\alpha_t = w_t L_t^G / Y_t^G$ is labor's share in goods production.

With intangibles nonrival, the production function for goods becomes:

$$y_t^G = A_t L_t^G.$$

Solow's residual no longer applies since the output elasticity of labor, which is now 1, no longer equals labor's share in goods production. Labor's share is determined by the magnitude of royalty payments to nonrival intangibles. This share may be large or small, as we will see in the models that follow, even though intangible capital does not enter the constant returns to scale production function.

Instead of Solow's residual we get a simpler formula for MFP growth:

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{y}_t^G}{y_t^G} - \frac{\dot{L}_t^G}{L_t^G}.$$

MFP growth will be faster when intangibles are treated as nonrival if the stock of intangibles grows faster than labor employed in the goods sector. Otherwise MFP will be faster when intangibles are treated as rival. This implication for productivity growth is why the distinction between rival and nonrival intangibles matters. Our assessment is that the nonrivalry of intangibles has been overlooked in standard national accounting methodology.

In addition to the measurement of MFP, our treatment of intangibles also matters for identifying what drives MFP. While nonrival intangibles don't enter the production function as do rival intangibles, they become a driving force behind MFP growth unlike rival intangibles.

The models that follow will fill in the critical details to establish these basic points more formally.

3 A Modeling Framework

Consider a closed economy with L_t workers at date t. Workers can either engage in production of goods or in the creation of intangibles. If L_t^I workers are engaged in the production of intangibles, the output of the intangibles sector is $A_t^I L_t^I$, where A_t^I is productivity in the intangibles sector. This output constitutes a gross contribution to the stock of intangibles. Acknowledging the possibility of depreciation, we have:

$$\dot{K}_t = A_t^I L_t^I - \delta K_t, \tag{1}$$

where $\delta \geq 0$ is the depreciation rate. Starting from date 0, the stock itself is:

$$K_t = e^{-\delta t} K_0 + \int_0^t e^{-\delta(t-s)} A_s^I L_s^I ds. \tag{2}$$

Since intangibles can't physically decay, we will focus on the case of $\delta = 0$. Productivity growth in the intangibles sector, like depreciation, will downweight investments made in the past relative to those made in the present within the stock K_t.[6]

The production sector makes a continuum Ω of differentiated goods, indexed by ω. For simplicity we treat Ω as measure 1. We assume that the individual goods are aggregated with a constant elasticity of substitution σ to form:

$$y_t^G = \left[\int_{\omega \in \Omega} b_t(\omega)^{1/\sigma} y_t(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)}, \tag{3}$$

[6] To make this point starkly, suppose productivity in intangibles grows at a constant rate $g^I > 0$ so that:

$$A_t^I = A_0^I e^{g^I t}.$$

Define a normalized intangible capital stock as:

$$\kappa_t = \frac{K_t}{A_t^I}.$$

This normalized capital stock, and its time derivative, will satisfy equations very similar to (2) and (1) with a depreciation rate of $g + \delta$. In particular:

$$\kappa_t = e^{-(\delta + g^I)t} \kappa_0 + \int_0^t e^{-(\delta + g^I)(t-s)} L_s^I ds$$

and

$$\dot{\kappa}_t = L_t^I - (\delta + g^I)\kappa_t.$$

where $y_t(\omega)$ is the quantity of good ω and $b_t(\omega)$ determines its importance to the aggregate.[7] If good ω sells at price $p_t(\omega)$, spending on the good will be:

$$p_t(\omega)y_t(\omega) = b_t(\omega)\left(\frac{p_t(\omega)}{P_t}\right)^{1-\sigma} Y_t^G,$$

where $Y_t^G = P_t y_t^G$ is total sales of the goods sector and the exact price index is:

$$P_t = \left[\int_{\omega\in\Omega} b_t(\omega)p_t(\omega)^{1-\sigma}d\omega\right]^{1/(1-\sigma)}. \tag{4}$$

For simplicity we assume labor is the only factor of production other than intangibles, with $l_t(\omega)$ denoting the number of workers making good ω at date t. Labor is rival in all activities (meaning that a worker can only be doing one thing at a time, either helping produce some good ω or creating intangibles). Exhaustion of the labor force implies:

$$L_t^I + L_t^G = L_t$$

where:

$$L_t^G = \int_{\omega\in\Omega} l_t(\omega)d\omega$$

A perfectly competitive labor market establishes a common wage w_t.

In this closed-economy setting we can let the wage be our numéraire, setting $w_t = 1$. It's easier to follow the economics, however, by leaving the wage to appear explicitly in the equations that follow.

We now consider two alternative specifications of production, depending on whether intangibles are rival or nonrival. Readers primarily interested in how to incorporate nonrival intangibles into a tractable accounting framework may want to skip to Section 2.2.

[7] We are free to normalize these good-level weights to have a mean of:

$$\bar{b}_t = \int_{\omega\in\Omega} b_t(\omega) = 1$$

3.1 The Case of Rival Intangibles

Output $y_t(\omega)$ of good ω at date t is given by the constant-returns-to-scale production function:

$$y_t(\omega) = a_t(\omega)F(l_t(\omega), k_t(\omega))$$

where $a_t(\omega)$ is exogenous productivity for good ω at date t and $k_t(\omega)$ is the stock of intangibles making good ω. Rivalry and exhaustion of intangibles implies that:

$$\int_{\omega \in \Omega} k_t(\omega) d\omega = K_t.$$

We assume that there is a perfectly competitive rental market for intangibles establishing a rental rate r_t. Cost minimization by producers of good ω implies a common capital labor ratio, which must equal the aggregate ratio:[8]

$$\frac{k_t(\omega)}{l_t(\omega)} = h(w_t/r_t) = \frac{K_t}{L_t^G}.$$

Letting $f(k/l) = F(1, k/l)$ we can write the production possibility frontier as:

$$\begin{aligned}
\int_{\omega \in \Omega} \frac{y_t(\omega)}{a_t(\omega)} d\omega &= \int_{\omega \in \Omega} f(k_t(\omega)/l_t(\omega)) l_t(\omega) d\omega \\
&= f(K_t/L_t^G) \int_{\omega \in \Omega} l_t(\omega) d\omega \\
&= F(L_t^G, K_t),
\end{aligned}$$

which depicts all (undominated) sequences of good-level output $\{y_t(\omega)\}_{\omega \in \Omega}$ that can be produced with aggregate factors of production L^G and K given good-level productivities $\{a_t(\omega)\}$.

We assume that the price of ω is a markup $m_t(\omega)$ on unit cost:

$$p_t(\omega) = m_t(\omega) \frac{c_t}{a_t(\omega)},$$

[8] The first order conditions for cost minimization imply:

$$\frac{w_t}{r_t} = \frac{F_l(l_t(\omega), k_t(\omega))}{F_k(l_t(\omega), k_t(\omega))} = g(k_t(\omega)/l_t(\omega)),$$

where the second inequality follows from the homogeneity of degree 0 of the derivatives of a function that's homogeneous of degree 1. The function h in the paper is g^{-1}.

where c_t is the cost of a bundle of inputs.[9] The nominal value of aggregate production Y_t^G is thus:

$$\begin{aligned} Y_t^G &= \int_{\omega \in \Omega} p_t(\omega) y_t(\omega) d\omega \\ &= c_t \int_{\omega \in \Omega} m_t(\omega) \frac{y_t(\omega)}{a_t(\omega)} d\omega. \end{aligned}$$

In the special case in which the markup $m_t(\omega)$ is independent of $y_t(\omega)/a_t(\omega)$ we have:

$$Y_t^G = \bar{m}_t c_t F(L_t^G, K_t),$$

where $\bar{m}_t$ is the mean of the markup. In what follows we present a convenient, albeit stringent, assumption that justifies this case.

Turning to the price index, we can substitute in the price of each good to get:

$$P_t = c_t \left[\int_{\omega \in \Omega} b_t(\omega) \left(\frac{m_t(\omega)}{a_t(\omega)} \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}.$$

To simplify this expression for P_t, and to justify our earlier simplification of Y_t^G, we introduce an assumption on the good-level weights that removes the correlation between markups and demand:

$$b_t(\omega) = m_t(\omega)^{\sigma} b_t'(\omega), \tag{5}$$

where the adjusted weights $b_t'(\omega)$ are independent of the markup and $a_t(\omega)$ while the markup

[9]Using the functions introduced above:

$$c_t = c(w_t, r_t) = \frac{w_t + r_t h(w_t/r_t)}{f(h(w_t/r_t))},$$

which is homogeneous of degree 1 in w and r.

is independent of $a_t(\omega)$.[10] Under assumption (5):

$$\begin{aligned} P_t &= c_t \left[\int_{\omega \in \Omega} b'_t(\omega) m_t(\omega) a_t(\omega)^{\sigma-1} d\omega \right]^{1/(1-\sigma)} \\ &= (\bar{m}_t \bar{b}'_t)^{1/(1-\sigma)} c_t \left[\int_{\omega \in \Omega} a_t(\omega)^{\sigma-1} d\omega \right]^{1/(1-\sigma)}, \end{aligned}$$

where:

$$\bar{b}'_t = \int_{\omega \in \Omega} b'_t(\omega) d\omega = \left[\int_{\omega \in \Omega} m_t(\omega)^{\sigma} d\omega \right]^{-1}.$$

Real output of the goods-producing sector, again assuming (5), is:

$$\begin{aligned} y_t^G &= \frac{\bar{m}_t c_t F(L_t^G, K_t)}{(\bar{m}_t \bar{b}'_t)^{1/(1-\sigma)} c_t \left[\int_{\omega \in \Omega} a_t(\omega)^{\sigma-1} d\omega \right]^{1/(1-\sigma)}} \\ &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)} \right)^{\sigma/(\sigma-1)} \left[\int_{\omega \in \Omega} a_t(\omega)^{\sigma-1} d\omega \right]^{1/(\sigma-1)} F(L_t^G, K_t) \\ &= A_t F(L_t^G, K_t), \end{aligned} \tag{6}$$

where $\bar{m}_t(\sigma)$ is the generalized mean of the markup, with exponent σ.[11] We have derived an

[10] The CES aggregator implies demand for variety ω of:

$$\begin{aligned} y_t(\omega) &= b_t(\omega) \left(\frac{p_t(\omega)}{P_t} \right)^{-\sigma} y_t^G \\ &= b_t(\omega) \left(\frac{m_t(\omega) c_t}{a_t(\omega) P_t} \right)^{-\sigma} y_t^G, \end{aligned}$$

so that demand responds inversly to independent variation in $m_t(\omega)$. Under assumption (5):

$$y_t(\omega) = b'_t(\omega) \left(\frac{c_t}{a_t(\omega) P_t} \right)^{-\sigma} y_t^G,$$

eliminating any dependence of demand on the markup.

[11] The generalized mean with exponent σ is:

$$\bar{m}_t(\sigma) = \left[\int_{\omega \in \Omega} m_t(\omega)^{\sigma} d\omega \right]^{1/\sigma},$$

hence:

$$\bar{m}_t(1) = \bar{m}_t$$

Due to assumption (5) we have

$$\bar{m}_t(\sigma) = (\bar{b}'_t)^{-1/\sigma}.$$

aggregate production function in which MFP is:

$$A_t = \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[\int_{\omega\in\Omega} a_t(\omega)^{\sigma-1} d\omega\right]^{1/(\sigma-1)}.$$

Consider the national accounts of this economy. Starting with the goods sector, revenue generated there goes to profits, payments to labor, and payments to intangibles:

$$Y_t^G = \Pi_t + w_t L_t^G + r_t K_t.$$

Continuing to invoke (5), these payments are split according to:

$$\Pi_t = \frac{\bar{m}_t - 1}{\bar{m}_t} Y_t^G$$

$$w_t L_t^G = \frac{\alpha}{\bar{m}_t} Y_t^G,$$

and

$$r_t K_t = \frac{1-\alpha}{\bar{m}_t} Y_t^G.$$

Nominal GDP is obtained by summing across sectoral revenues:

$$\begin{aligned} Y_t &= Y_t^G + Y_t^I \\ &= \Pi_t + w_t L_t^G + r_t K_t + w_t L_t^I. \end{aligned}$$

We can also express GDP by summing across payments to factors (and adding in profits):

$$Y_t = \Pi_t + Y_t^L + Y_t^K,$$

where labor income is:

$$Y_t^L = w_t L_t = w_t L_t^G + w_t L_t^I,$$

and the returns to intangibles are:

$$Y_t^K = r_t K_t$$

3.2 The Case of Nonrival Intangibles

We now consider the alternative approach, which treats intangibles as nonrival technologies, no longer included within the constant returns to scale production function. In this case output of good ω at date t is:

$$y_t(\omega) = a_t(\omega) l_t(\omega), \tag{7}$$

where $a_t(\omega)$ is the technology used to make good ω at date t. The wage, common to all producers, remains w_t. We immediately have the production possibility frontier:

$$\int_{\omega \in \Omega} \frac{y_t(\omega)}{a_t(\omega)} d\omega = L_t^G.$$

Mapping from the aggregate stock of intangibles, given by (2), to the set of production technologies requires a theory of technological change. We treat $A_t^I L_t^I$ as governing the arrival of new ideas about production technologies, each one suitable for producing one or many varieties $\omega \in [0, 1]$.[12] The set of ideas that is available depends on past investments in intangibles, as embodied in K_t. We denote the technological efficiency of a particular idea for good ω by $a(\omega)$, which is the productivity of a worker using the idea to produce ω. For any given good ω we can rank all the ideas that are available at date t:

$$a_t^{(1)}(\omega) \geq a_t^{(2)}(\omega) \geq a_t^{(3)}(\omega) \geq ....$$

While the efficiency of an idea is fixed, its position in this ranking will typically evolve over time, hence the t subscript.

We assume that investors in intangibles acquire property rights to their ideas, say in the form of patent protection, which allows them to choose who can use the idea. For much of what we do, these property rights have value only for the owner of the best idea.

[12] While a single idea can be used in producing a countable number of different varieties, to avoid dealing with strategic investment in intangibles we assume that this set of varieties is small relative to the total (has measure zero).

To facilitate comparison to the case of rival intangibles, we treat investment in nonrival intangibles as being separated from production, as in Arrow (1962). The owner of the best idea for producing ω, with efficiency $a_t^{(1)}(\omega)$, licenses the technology to producers of good ω, charging a royalty per unit produced. For producers to be willing to adopt the idea, the royalty rate, expressed relative to production cost, must satisfy an upper bound of:

$$R_t(\omega) \leq \frac{a_t^{(1)}(\omega)}{a_t^{(2)}(\omega)} - 1 = R_t^{\max}(\omega).$$

A producer is willing to pay the royalty rate $R_t(\omega)$ rather than to be stuck using the second best technology, or anything worse than that, even for free.[13] Imposing this bound of $R_t^{\max}(\omega)$, the production technology used in equilibrium will be:

$$a_t(\omega) = a_t^{(1)}(\omega).$$

Given the royalty rate, and any additional markup of $m_t(\omega)$ over total unit costs (inclusive of royalties) the producer sets a price of:

$$p_t(\omega) = m_t(\omega)\left(1 + R_t(\omega)\right)\frac{w_t}{a_t(\omega)}. \tag{8}$$

Using (8), the nominal value of aggregate goods production is:

$$\begin{aligned} Y_t^G &= \int_{\omega\in\Omega} p_t(\omega) y_t(\omega) d\omega \\ &= w_t \int_{\omega\in\Omega} m_t(\omega)\left(1 + R_t(\omega)\right)\frac{y_t(\omega)}{a_t(\omega)} d\omega. \end{aligned}$$

[13] Suppose the producer can use the second best technology for free. In that case its cost is:

$$c_t^{(2)}(\omega) = \frac{w_t}{a_t^{(2)}(\omega)}.$$

By adopting the best technology, paying a royalty rate $R_t(\omega)$, its unit cost is:

$$c_t(\omega) = \frac{w_t}{a_t^{(1)}(\omega)}\left(1 + R(t)\right) \leq \frac{w_t}{a_t^{(1)}(\omega)}\frac{a_t^{(1)}(\omega)}{a_t^{(2)}(\omega)} = c_t^{(2)}(\omega).$$

If the markup is independent of $y_t(\omega)/a_t(\omega)$, we have:

$$\begin{aligned} Y_t^G &= \bar{m}_t w_t \int_{\omega\in\Omega} (1+R_t(\omega)) \frac{y_t(\omega)}{a_t(\omega)} d\omega \\ &= \bar{m}_t w_t (1+\beta_t) L_t^G, \end{aligned} \tag{9}$$

where β_t is the ratio of royalty payments (for intangible capital) to wage payments:

$$\beta_t = \frac{\int_{\omega\in\Omega} R_t(\omega) \frac{w_t}{a_t(\omega)} y_t(\omega) d\omega}{w_t L_t^G} = \int_{\omega\in\Omega} s_t^l(\omega) R_t(\omega) d\omega, \tag{10}$$

where:

$$s_t^l(\omega) = l_t(\omega)/L_t^G$$

is the share of labor in the goods producing sector allocated to the production of good ω.

To advance further, we make the same assumptions on demand as in the case of rival tangibles, resulting in the price index (4). Substituting in (8), invoking assumption (5) as in the case of rival intangibles, and also assuming that the markup is independent of the royalty rate, the price index becomes:

$$\begin{aligned} P_t &= w_t \left[\int_{\omega\in\Omega} b_t(\omega) \left(\frac{m_t(\omega)(1+R_t(\omega))}{a_t(\omega)} \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \\ &= (\bar{m}_t \bar{b}_t')^{1/(1-\sigma)} w_t \left[\int_{\omega\in\Omega} \left(\frac{1+R_t(\omega)}{a_t(\omega)} \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}. \end{aligned} \tag{11}$$

Real output of the goods producing sector is thus:

$$\begin{aligned} y_t^G &= \frac{\bar{m}_t w_t (1+\beta_t) L_t^G}{(\bar{m}_t \bar{b}_t')^{1/(1-\sigma)} w_t \left[\int_{\omega\in\Omega} \left(\frac{1+R_t(\omega)}{a_t(\omega)} \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}} \\ &= A_t L_t^G, \end{aligned}$$

where MFP is:

$$A_t = \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)} \right)^{\sigma/(\sigma-1)} \left[\int_{\omega\in\Omega} \left(\frac{a_t(\omega)}{1+R_t(\omega)} \right)^{\sigma-1} d\omega \right]^{1/(\sigma-1)} (1+\beta_t). \tag{12}$$

While this productivity expression lacks parsimony, a key message comes through from the simple aggregate production function. Unlike in the case of rival intangibles, real output can expand in proportion to labor, holding fixed the stock of intangibles. This property is the fundamental insight of treating intangibles as nonrival.

We now turn to whether it's optimal for the owner of the best idea to set the royalty rate at the upper bound. Recall that for the CES aggregator, the quantity demanded of good ω is:

$$y_t(\omega) = b_t(\omega) \left(\frac{p_t(\omega)}{P_t} \right)^{-\sigma} y_t^G$$

Royalty revenue for good ω is thus:

$$\begin{aligned} R_t(\omega) \left(\frac{w_t}{a_t(\omega)} \right) y_t(\omega) &= R_t(\omega) \left(\frac{w_t}{a_t(\omega)} \right) b_t(\omega) \left(\frac{p_t(\omega)}{P_t} \right)^{-\sigma} y_t^G \\ &= R_t(\omega) \left(\frac{w_t}{a_t(\omega)} \right) b_t(\omega) \left(\frac{m_t(\omega)\left(1 + R_t(\omega)\right) \frac{w_t}{a_t(\omega)}}{P_t} \right)^{-\sigma} y_t^G \\ &= R_t(\omega)\left(1 + R_t(\omega)\right)^{-\sigma} \left(b_t(\omega) \left(\frac{w_t}{a_t(\omega)} \right)^{1-\sigma} \left(\frac{m_t(\omega)}{P_t} \right)^{-\sigma} y_t^G \right) . \end{aligned} \quad (13)$$

The owner's problem therefore reduces to maximizing $R_t(\omega)\left(1 + R_t(\omega)\right)^{-\sigma}$ subject to the upper bound on $R_t(\omega)$. We assume that the owner takes $m_t(\omega)$ as given. The resulting optimal royalty rate is $R_t^{\max}(\omega)$ for $\sigma \leq 1$ and for $\sigma > 1$:

$$R_t(\omega) = \min \left\{ \frac{a_t^{(1)}(\omega)}{a_t^{(2)}(\omega)}, \frac{\sigma}{\sigma - 1} \right\} - 1, \quad (14)$$

where the second term in the minimization kicks in for a so-called drastic innovations.[14]

[14]To see why, let:

$$f(R) = R(1+R)^{-\sigma},$$

with derivative:

$$f'(R) = (1+R)^{-\sigma} - \sigma R(1+R)^{-\sigma-1}.$$

For $\sigma \leq 1$ the derivative is always positive, so the optimal R is $R^{\max}$. If $\sigma > 1$, the first order condition is:

$$(1+R)^{-\sigma} = \sigma R(1+R)^{-\sigma-1},$$

which implies:

$$R = \frac{1}{\sigma - 1}.$$

By our definition (10), aggregate royalties can be expressed as:

$$\rho_t K_t = \beta_t w_t L_t^G = \left(\int_{\omega \in \Omega} s_t^l(\omega) R_t(\omega) d\omega \right) w_t L_t^G.$$

A doubling of L_t^G (holding fixed intangible capital and the allocation of labor across varieties) doubles payments to intangibles. Each nonrival idea is used to produce twice as many units of output, which is possible due to the nonrivalry of these technologies.

Expressed relative to payments to labor, payments to intangible capital are:

$$\beta_t = \frac{\rho_t K_t}{w_t L_t^G} = \int_{\omega \in \Omega} s_t^l(\omega) R_t(\omega) d\omega.$$

This ratio will be close to zero if the efficiency of the best and next best ideas are typically similar to each other. On the other hand if the efficiency gaps are large and if demand is inelastic payments to intangibles can be arbitrarily large relative to payments to labor. With elastic demand ($\sigma > 0$) and assuming a constant markup ($m_t(\omega) = \bar{m}_t$), we have a tighter upper bound:

$$\beta_t \leq \frac{1}{\sigma - 1},$$

which is reached if all innovations are drastic.

Let's reconsider the national accounts for this economy. For the goods producing sector:

$$Y_t^G = \Pi_t + w_t L_t^G + \rho_t K_t,$$

just like for the case of rival intangibles except that r_t is replaced by ρ_t. Assumption (5) implies pure profit is:

$$\Pi_t = \frac{\bar{m}_t - 1}{\bar{m}_t} Y_t^G,$$

with payments to labor:

$$w_t L_t^G = \frac{1}{\bar{m}_t \left(1 + \beta_t\right)} Y_t^G,$$

and payments to intangibles:

$$\rho_t K_t = \frac{\beta_t}{\bar{m}_t \left(1 + \beta_t\right)} Y_t^G.$$

Again, we require $R \leq R^{\max}$.

These shares are the same as for the case of rival intangibles except that the output elasticity of rival intangible capital $1-\alpha_t$ is replaced by royalty payments as a share of overall factor payments, $\beta_t/(1+\beta_t)$, which depends on endogenous royalty rates. To say more we need to consider specific cases.

Just as in the case of rival intangibles, we get nominal GDP by summing across sectoral revenues:

$$Y_t = Y_t^G + Y_t^I.$$

We've made no change to our treatment of investment in intangibles Y^I, so this part of the accounting is identical to the case of rival intangibles.

We now turn to some prominent models that flesh out this structure. At the level of individual varieties, we want to see how $a_t^{(1)}(\omega)$ and $R_t(\omega)$ jointly evolve with K_t. In the aggregate, we want to see how A_t and the royalty share evolve with K_t.

3.2.1 Grossman and Helpman Model

The "quality ladders" model of Grossman and Helpman (1991) has played a prominent role in the "new growth theory." It beautifully illustrates the points sketched out above.

In quality ladders, ideas for any good ω arrive as a Poisson process at rate $A_s^I L_s^I$ for $s \in [0,t]$. Thus, as of date t, the number of ideas $N_t(\omega)$ for producing good ω is distributed Poisson with parameter K_t:

$$\Pr\left[N_t(\omega) = N\right] = \frac{e^{-K_t} K_t^N}{N!}.$$

We'll follow Grossman and Helpman in assuming $\delta = 0$, which simplifies the exposition. We'll be somewhat more general in allowing $\sigma \geq 1$, whereas they assume $\sigma = 1$. We also allow for a markup of price over marginal cost, on top of the royalty rate.

Ideas represent equally spaced steps on the quality ladder, so that:

$$\frac{a_t^{(1)}(\omega)}{a_t^{(2)}(\omega)} = \lambda,$$

where $a_t^{(1)}(\omega)$ is the efficiency of the last idea and $a_t^{(2)}(\omega)$ the efficiency of the previous one. Each new idea is the same percentage advance over the one that proceeded it for producing the same good. Starting from $a_0^{(1)}(\omega) = 1$, we have:

$$a_t(\omega) = a_t^{(1)}(\omega) = \lambda^{N_t(\omega)}. \tag{15}$$

Several key simplifications follow. For $\lambda \leq \sigma/(\sigma-1)$, we have $R_t(\omega) = R_t = \lambda - 1$.[15] It follows from (10) that $\beta_t = \lambda - 1$ as well. We can therefore express revenue of the goods sector (9) as:

$$Y_t^G = \bar{m}_t w_t \left(1 + \beta_t\right) L_t^G = \bar{m}_t \lambda w_t L_t^G.$$

Similarly, the price index (11) reduces to:

$$\begin{aligned} P_t &= (\bar{m}_t \bar{b}_t')^{1/(1-\sigma)} w_t \left[\int_{\omega\in\Omega} \left(\frac{\lambda}{a_t(\omega)}\right)^{1-\sigma} d\omega\right]^{1/(1-\sigma)} \\ &= (\bar{m}_t \bar{b}_t')^{1/(1-\sigma)} \lambda w_t \left[\int_{\omega\in\Omega} a_t(\omega)^{\sigma-1} d\omega\right]^{1/(1-\sigma)} \end{aligned}$$

Thus, the aggregate production function is:

$$y_t^G = A_t L_t^G,$$

where MFP residual (12) reduces to:

$$A_t = \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[\int_{\omega\in\Omega} a_t(\omega)^{\sigma-1} d\omega\right]^{1/(\sigma-1)}.$$

We can simplify MFP, first by substituting in (15):

$$A_t = \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[\int_{\omega\in\Omega} \lambda^{(\sigma-1)N_t(\omega)} d\omega\right]^{1/(\sigma-1)}.$$

[15] Grossman and Helpman don't introduce royalties, instead modeling individual firms that innovate and then produce. Our approach, which separates investment in intangibles from production, yields the same aggregate outcomes.

Assuming $b_t'(\omega)$ is independent of $N_t(\omega)$ and exploiting the moment generating function of the Poisson, we get:

$$\begin{aligned} A_t &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[\int_{\omega\in\Omega} e^{(\ln\lambda)(\sigma-1)N_t(\omega)}d\omega\right]^{1/(\sigma-1)} \\ &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[\sum_{x=0}^{\infty} e^{(\ln\lambda)(\sigma-1)x}\frac{e^{-K_t}K_t^x}{x!}\right]^{1/(\sigma-1)} \\ &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \left[e^{K_t\left(e^{\ln\lambda(\sigma-1)}-1\right)}\right]^{1/(\sigma-1)} \\ &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} e^{\frac{\lambda^{\sigma-1}-1}{\sigma-1}K_t}. \end{aligned}$$

The quality ladder model thus delivers an explicit link from the stock of intangibles to MFP. Productivity growth, assuming a constant distribution of markups, is thus:

$$\frac{\dot{A}_t}{A_t} = \frac{\lambda^{\sigma-1}-1}{\sigma-1}\dot{K}_t,$$

for $\sigma > 1$.[16] Constant productivity growth is sustained with a constant flow of intangible investment. If real investment rises, either due to growth in L^I or in A^I, growth would accelerate.

The quality ladders model also yields an explicit decomposition of revenue in the goods sector:

$$Y_t^G = \Pi_t + w_t L_t^G + \rho_t K_t,$$

where:

$$\begin{aligned} \Pi_t &= \frac{\bar{m}_t - 1}{\bar{m}_t}Y_t^G, \\ w_t L_t^G &= \frac{1}{\bar{m}_t\lambda}Y_t^G \\ \rho_t K_t &= \frac{\lambda-1}{\bar{m}_t\lambda}Y_t^G. \end{aligned}$$

The share of revenue paid to intangibles rises with the innovative step size λ.

[16] In the limit as $\sigma \to 1$, l'Hopital's rule yields:

$$\frac{\dot{A}_t}{A_t} = (\ln\lambda)\,\dot{K}_t.$$

3.2.2 EK Model

Whereas every new idea represents an inventive step in the quality ladders model, we can also conceive of technological change as a process of trial and error, with new ideas often failing to improve on those that came before. We go back to a foundation in random search first proposed by Evenson and Kislev (1976) and extended by Bental and Peled (1996) and Kortum (1997). This random search formulation has proved to be particularly useful when expanded to an international setting as in: (i) international technology diffusion and growth (EK, 1999), international trade (EK, 2002), and trade in intangible services (EK, 2019). We will turn to the international implications later, here continuing to work with a closed economy.

The arrival of ideas is exactly as in the quality ladders model. Unlike quality ladders, each idea yields a random level of labor efficiency $a(\omega)$, drawn from a Pareto distribution with parameter θ. (A larger value of θ leads to the efficiency of ideas being less dispersed.) Following the arguments in EK (2019b), the number of ideas that arrive by date t for producing good ω with efficiency greater than z is distributed Poisson with parameter:

$$\mu_t(z) = K_t z^{-\theta}.$$

This process for the arrival of ideas of various efficiency levels gives us the cumulative distribution for the efficiency level $a_t^{(1)}(\omega)$ of the best technology for producing good ω. The probability that no idea with efficiency above z has arrived by date t is:

$$\Pr\left[a_t^{(1)}(\omega) \leq a\right] = e^{-K_t a^{-\theta}},$$

the Fréchet distribution. While realizations will differ, this distribution is common across all varieties.

Unlike quality ladders, in this model the royalty rate $R_t(\omega)$ is random and is positively correlated with the level of technology $a_t(\omega) = a_t^{(1)}(\omega)$. A breakthrough in technology that raises $a_t(\omega)$ also opens up a gap between $a_t^{(1)}(\omega)$ and $a_t^{(2)}(\omega)$, which raises the royalty rate. Accounting for this correlation requires extra care in the derivations.

In EK (2019b) we show that royalty payments relative to wage payments in the aggregate are:

$$\frac{\rho_t K_t}{w_t L_t^G} = \beta_t = \frac{1}{\theta}.$$

While royalty rates vary across varieties, this ratio remains constant. It is decreasing in θ since higher θ implies smaller gaps, on average, between the efficiency of the best and second-best ideas for any given good.

Revenue of the goods sector (9) is therefore:

$$Y_t^G = \bar{m}_t w_t \left(1+\beta_t\right) L_t^G = \bar{m}_t \frac{1+\theta}{\theta} w_t L_t^G.$$

Using distributional results from EK (2019b), we can integrate the price index (11) over the joint distribution of $R_t(\omega)$ and $a_t(\omega)$ to get:

$$P_t = \gamma(\bar{m}_t \bar{b}_t')^{1/(1-\sigma)} w_t K_t^{-1/\theta},$$

where:

$$\gamma = \left(\Gamma\left(2-\frac{\sigma-1}{\theta}\right)\left(1+\frac{\sigma-1}{\theta-\sigma+1}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\right)\right)^{-1/(\sigma-1)}.$$

The goods-sector production function is thus:

$$\begin{aligned} y_t^G &= \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \frac{\frac{1+\theta}{\theta} w_t L_t^G}{\gamma w_t K_t^{-1/\theta}} \\ &= A_t L_t^G, \end{aligned}$$

where MFP is:

$$A_t = \left(\frac{\bar{m}_t}{\bar{m}_t(\sigma)}\right)^{\sigma/(\sigma-1)} \frac{1+\theta}{\theta\gamma} K_t^{1/\theta}.$$

Productivity growth, assuming a constant distribution of markups, is thus:

$$\frac{\dot{A}_t}{A_t} = \frac{1}{\theta}\frac{\dot{K}_t}{K_t}.$$

This result appears fundamentally different from quality ladders, since here the stock of intangibles must grow at a constant rate to generate constant productivity growth. Without having taken a stand on productivity in the intangibles sector, however, this apparent difference has little quantitative bite.

The accounts for this model are unchanged from above if we simply replace β_t by $1/\theta$.

3.3 Assessment

What is the consequence of treating the stock of intangibles as rival versus nonrival? In answering this question, we want to look at the same economy through two different lenses: the models of Sections 2.1 and 2.2. We'll treat measured outcomes Y_t^G, $w_t L_t^G$, $w_t L_t^I$, and Y_t as being the same in these two scenarios. We'll also treat the distribution of markups as the same (as reflected in $\bar{m}_t$ and $\bar{m}_t(\sigma)$) and hence the return to intangibles $r_t K_t = \rho_t K_t$ as well. Finally, we'll treat the real outcomes of A_t^I, K_t, and y_t^G as being the same. What's left is MFP for goods production, A_t, and that's where it matters which lens we look through.

4 Conclusion

Intangible are typically durable and nonrival. While national accounting procedure has evolved to integrate this first property, it can be inconsistent with the second. We argue that nonrival intangibles have implications for measuring both the level of intangible investment and the growth of productivity. We lay out a prototype modeling framework to show how nonrivalry could, at least in principle, be integrated into national accounting procedures.

An important next step is to extend this analysis to a global setting. There we confront Hill's point that intangibles "have no physical dimensions or spatial co-ordinates of their own." They can be used in any number of countries at the same time and their returns may show up anywhere. These properties pose challenges for modeling but also for measurement, as highlighted in Berry et. al. (2020).

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