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Price-rent Ratios and Expected Capital Gains–
A Hedonic Spatio-temporal Approach

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Price-rent ratios and expected capital gains – A hedonic spatio-temporal approach

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Abstract

The price-rent ratio is one of the most important measures for monitoring the housing market. This paper outlines and adopts a hedonic spatio-temporal methodology for estimating quality-adjusted price-rent ratios for apartments in 21 major cities in Germany. With the user-cost equilibrium condition it is subsequently possible to derive estimates of the cross-section of expected real capital gains. In addition, quality-adjusted property price and rental indices are computed at the city-level. Using this new hedonic method applied to prices and rents over the period 2014Q2 – 2018Q1, we find a large degree of heterogeneity across cities and time. These findings deliver deep insights into the dynamics of the German housing market and have important implications for housing investment and urban planning.

Keywords Housing market · Price-rent ratios · User cost · Capital gains · Spatio-temporal hedonic model

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1 Introduction

The relationship between sales prices and rents of residential real-estate properties has been one of the most popular measures for monitoring the housing market. Special attention attracted high price-rent ratios, since it potentially signals an overvaluation, or bubble, in sales prices. The implied consequence is that the affordability of decent housing decreases in proportion to a decline in the ratio (Lee and Park, 2018). Governments employ various policies to cool off booms in sales prices and, especially, in rents during periods with high price-rent ratios.

In this article, we outline and adopt a hedonic spatio-temporal modeling approach based on semi-parametric smoothing splines which uses sales prices and rents in a joint model. We study the determination of the price-rent ratio for apartments in 21 major German cities over the period 2014Q2 – 2018Q1. In a second step we apply the user-cost equilibrium condition to investigate the cross-section of expected capital gains. In addition, we construct quality-adjusted property price and rental indices at the city-level. This provides deep insights into the dynamics of the German housing market and has important implications for the housing policy of local authorities and strategies of real-estate companies and other market participants.

Our results show that there is a degree of heterogeneity across cities and time. We find price-rent ratios between 15.0 and 33.2, and expected real capital gain between 5.9% to 9.7% in 2018Q1. The increase in the sales price was between 13.5% and 60.5% over the sample period, while the increase in the rents was more moderate between 10.7% and 48.7%.

The remainder of this paper is organized as follows. In Section 2, we describe our data, the spatio-temporal model that we use for estimating the price-rent ratios, and the construction principle of quality-adjusted hedonic property price and rental indices. In Section 3, we present the user-cost approach and the adopted procedure to estimate capital gains. In Section 4 we exhibit our empirical findings for the German housing market. Section 5 concludes the paper.

2 Spatio-temporal estimation of quality-adjusted price-rent ratios

Data

The data availability for Germany is different compared to other highly developed countries. Especially on the local level, it is hard to find micro-level sources that allow for an analysis over longer time-periods. German time series are typically short, cover only a few (or single) locations, or contain asking prices (Kholodilin and Michelsen, 2017).

Our hedonic data set covers the German real-estate market in 21 major cities between 2014Q2 and 2018Q1, consisting after data cleaning of 504,797 asking sales and 1,111,890 asking rents for apartments, provided by Bulgienwesa AG and based on announcements on Immoscout24.de.[2,3] Note that 66.8% of the sales and 55.1% of the rental observations belong to locations classified as A-cities, i.e. to the most (internationally) important markets which feature excellent real estate market conditions with an annual turnover in each city over 2.5 percent of the national market (Kholodilin and Michelsen, 2017).[4] B-cities are nationally and/or regionally important and have an annual turnover volume of over 1.5 percent of the market.

The data set includes information for each individual apartment on the advertised rent per square meter per month/purchasing price per square meter, the year and quarter when the advertisement was online, unit-level structural attributes (for example, year of construction, floor of unit, or number of bathrooms), locational attributes (for example,

[2]To avoid an overwhelming number of results for too many cities, we restrict our analysis in this paper to the 21 major cities in Germany. We also have access to data for further 22 regional and 84 local centers. Empirical results for those cities are available on request.

[3]We remove the outliers with extreme prices and rents per square meter (the 1% highest). We also filter out some observations with implausible characteristics and repeatedly advertised apartments. In the latter case, we keep only the most recent observation in our records.

[4]There are seven A-cities: Berlin, Cologne, Düsseldorf, Frankfurt on the Main, Hamburg, Munich, and Stuttgart. The fourteen B-cities are: Bochum, Bonn, Bremen, Dortmund, Dresden, Duisburg, Essen, Hanover, Karlsruhe, Leipzig, Mannheim, Münster, Nuremberg, and Wiesbaden.

population density, gastronomy supply, or public transport), as well as exact location (longitudes and latitudes). Table 1 presents the definition and major descriptive statistics of the variables.

The model introduced in the next paragraph will be estimated individually for each of the 21 cities. Table 2 summarizes the number of observations available per city.

Spatio-temporal model

In standard hedonic modeling, the log of the sales price or rent, P, is regressed on a vector of covariates which describe structural and locational attributes of the individual dwellings, and time dummies. In this paper we extend the Local Regression Model (LRM) introduced by Clapp (2004) and adopt it to our purpose, the estimation of the quality-adjusted price-rent ratio over time.

Clapp (2004) proposes a semi-parametric approach of great flexibility which allows to identify space-time asymmetries missed by other models. The LRM is similar to the standard hedonic model, except that a flexible function is introduced for the value of space and time:

$$y_{it} = \log(P_{it}) = \boldsymbol{X}_i\beta + f(\boldsymbol{Z}_i) + \varepsilon_{it}, \tag{1}$$

where ε_{it} is i.i.d. noise that is assumed to be normally distributed; $\boldsymbol{Z}_i$ is the three-dimensional vector of latitude, longitude, and time; and $\boldsymbol{X}_i$ is the vector of unit-specific and location-specific characteristics. Clapp (2004) estimates the LRM with a local-polynomial smoother based on procedures proposed by Robinson (1988) and Stock (1989). A similar approach with a local-constant smoother was used more recently by Zhu et al. (2019). We instead apply an estimation method based on thin-plate regression splines as used, for example, in Peterl (2017), Hill and Scholz (2018) or Hill et al. (2018). The reasons for this choice are: (i) We can directly model the mean $\mu_i = \mathbb{E}(P_i)$ of a response variable of interest which comes from some exponential family distribution. Here, the underlying model is a Generalized Additive Model (GAM) of the form

$$g(\mu_i) = \sum_k f_k(v_{ik}) + \boldsymbol{X}_i\beta \tag{2}$$

where f_k are unknown nonparametric functions on (higher-dimensonal) covariates v_k and g is a known link-function (in our case the natural logarithm). This approach is more flexible because it allows us to model prices and rents being Gamma distributed and thus allowing for heavier and more realistic tails in there distributions. Furthermore, we can avoid the back-transformation of the predictor from the log-scale which usually requires a precise estimate of the error variance (see, for example, Wooldrige (2012) for the OLS case). (ii) The free statistical software R (R Core Team, 2018) provides a state-of-the-art implementation of GAM in the package `mgcv`. For more details on GAM's, see, for example, Wood (2017). More information of parameter choices and possible robustness checks for applications of GAM's in a housing context, can bee found, for example, the appendix A.3 in Hill and Scholz (2018). (iii) The proposed estimation procedure used, for example, in Clapp (2004) and Zhu et al. (2019) is not the state-of-the-art in applied statistics. Nielsen and Sperlich (2005) introduce a feasible cross-validation procedure for the smooth backfitting procedure for local-polynomial smoothers applied in the context of estimating GAM's which is more efficient, robust and easier to calculate than competing methods. Unfortunately, their R package is not yet available.

For our model for sales prices and rents, we extend the approach proposed in Peterl (2017) (without a spatio-temporal component) and applied to housing data from Sydney, Australia. Note that we model sales prices and rents jointly, what allows us to derive a simple estimate for quality-adjusted price-rent ratios not depending on individual characteristics. Our is defined as:

$$\begin{aligned}\eta_i &= \beta_0^{\text{rent}} D_i + \beta_0^{\text{sale}}(1 - D_i) + f_1^{\text{rent}}(Q_i)D_i + f_1^{\text{sale}}(Q_i)(1 - D_i) \\ &+ f_2(YOC_i) + f_3(AREA_i) + \sum_{k=1}^{5} f_{k+3}(LONG_i, LAT_i)Y_{ki} + \boldsymbol{X}_i\beta \end{aligned} \tag{3}$$

Table 1: Definition and descriptive statistics of available variables

		Sales Sample		Rents Sample	
Variable	Description	Mean	Std. Dev.	Mean	Std. Dev.
$PRICE$	rent per sqm per month/purchasing price per sqm	3376.8	1830.5	9.63	3.88
Structural Attributes					
$AREA$	Area of the unit (m^2)	85.0	35.5	72.2	29.6
YOC	Year of construction	1969.9	41.6	1961.4	37.8
$TYPE\_1$	1 if unit is on the ground floor	0.109	0.312	0.104	0.305
$TYPE\_2$	1 if unit is on the top floor	0.091	0.287	0.102	0.303
$TYPE\_3$	1 if unit is a loft	0.006	0.077	0.004	0.062
$TYPE\_4$	1 if unit is a maisonette	0.052	0.223	0.033	0.178
$TYPE\_5$	1 if unit is a penthouse	0.027	0.162	0.010	0.101
$TYPE\_6$	1 if unit is a terrace apartment	0.021	0.145	0.015	0.122
$TYPE\_7$	1 if unit is a storey apartment	0.527	0.499	0.552	0.497
$TYPE\_8$	1 if unit is a mezzanine	0.028	0.164	0.026	0.160
$TYPE\_9$	1 if unit is a souterrain	0.003	0.057	0.006	0.077
$TYPE\_10$	1 if unit is of an other type/NA	0.019	0.135	0.014	0.117
$TYPE\_11$	1 if unit is of unknown type	0.117	0.321	0.134	0.340
$MKAT\_1$	1 for renovation between 1992 and 1997	0.015	0.121	0.010	0.099
$MKAT\_2$	1 for renovation between 1997 and 2002	0.029	0.167	0.019	0.136
$MKAT\_3$	1 for renovation between 2002 and 2007	0.027	0.164	0.021	0.144
$MKAT\_4$	1 for renovation between 2007 and 2012	0.051	0.221	0.070	0.256
$MKAT\_5$	1 for renovation between 2012 and 2017	0.124	0.330	0.202	0.401
$MKAT\_6$	1 for renovation before 1992 or never/NA	0.753	0.431	0.678	0.467
$EKAT\_1$	1 if unit is on ground floor	0.102	0.302	0.103	0.304
$EKAT\_2$	1 if unit is on 1st or 2nd floor	0.371	0.483	0.441	0.497
$EKAT\_3$	1 if unit is on 3rd or 4th floor	0.216	0.411	0.253	0.435
$EKAT\_4$	1 if unit is on 5th to 10th floor	0.069	0.254	0.072	0.259
$EKAT\_5$	1 if unit is on 11th floor or higher	0.005	0.073	0.006	0.077
$EKAT\_6$	1 if unit is on unknown floor/NA	0.237	0.425	0.124	0.330
$LIFT\_1$	1 if unit has a lift	0.437	0.496	0.305	0.461
$LIFT\_2$	1 if unit has no lift	0.511	0.500	0.638	0.481
$LIFT\_3$	1 if unknown lift information/NA	0.052	0.221	0.057	0.231
BK_1	1 if unit has a built-in kitchen	0.317	0.465	0.407	0.491
BK_2	1 if unit has no built-in kitchen	0.604	0.489	0.529	0.499
BK_3	1 if unknown built-in kitchen information/NA	0.079	0.269	0.064	0.244
$BALC\_1$	1 if unit has a balcony	0.742	0.437	0.671	0.470
$BALC\_2$	1 if unit has no balcony	0.230	0.421	0.305	0.461
$BALC\_3$	1 if unknown balcony information/NA	0.028	0.164	0.024	0.154
$GARD\_1$	1 if unit has a garden	0.222	0.416	0.172	0.377
$GARD\_2$	1 if unit has no garden	0.686	0.464	0.743	0.437
$GARD\_3$	1 if unknown garden information/NA	0.092	0.289	0.085	0.279
$BATH\_1$	1 if unit has 1 bathroom	0.565	0.496	0.624	0.484
$BATH\_2$	1 if unit has 2 bathrooms	0.160	0.367	0.077	0.267
$BATH\_3$	1 if unit has 3 or more bathrooms	0.007	0.081	0.002	0.042
$BATH\_4$	1 if unknown bathroom information/NA	0.268	0.443	0.297	0.457
GWC_1	1 if unit has a guest toilet	0.287	0.452	0.169	0.375
GWC_2	1 if unit has no guest toilet	0.704	0.457	0.825	0.380
GWC_3	1 if unknown guest toilet information/NA	0.009	0.095	0.006	0.079

Table 1: *Cont.*

Variable	Description	Sales Sample		Rents Sample	
		Mean	Std. Dev.	Mean	Std. Dev.
Locational Attributes					
GS^*	gastronomy supply in vicinity	1		1	
PSV^*	primary schools in vicinity	4		4	
KGV^*	kindergarten in vicinity	3		3	
PD^{**}	population density	7		7	
PP^{**}	purchasing power	6		6	
QRA^{**}	quality of residential area	4		4	
RSF^{***}	retail supply for food	2		2	
BD^{****}	building dominance	10		10	
PTD	public transport density	1653.5	1258.9	1422.0	1078.1
$LONG$	longitude	10.880	2.531	10.254	2.641
LAT	latitude	51.323	1.564	51.292	1.389

Notes: (1) There are 504,797 sales and 1,111,890 rents observations for 21 major cites in Germany. (2) Most of the locational attributes are categorized with Jenks natural break method ($^* = 5$ groups, $^{**} = 7$ groups, $^{***} = 9$ groups, $^{****} = 15$ groups) ordered from poorest to excellent/ low to high. Here we report the category with the largest number of obs. (3) PTD is the numeric proportional sum of public transport points in vicinity, best in urban areas.

Table 2: Number of observations per city

City name	Category	Number of sales	Number of rents
Berlin	A	172692	238818
Cologne	A	24162	63998
Düsseldorf	A	19692	68361
Frankfurt on the Main	A	19749	60358
Hamburg	A	34133	75645
Munich	A	49592	77441
Stuttgart	A	17015	28444
Bochum	B	5798	22377
Bonn	B	7905	26620
Bremen	B	7253	20400
Dortmund	B	11785	37527
Dresden	B	23720	75218
Duisburg	B	11422	40364
Essen	B	14526	55003
Hanover	B	12620	31036
Karlsruhe	B	4318	11790
Leipzig	B	27481	102161
Mannheim	B	9886	17416
Münster	B	5669	13427
Nuremberg	B	16081	21739
Wiesbaden	B	9298	23747

where η_i is the linear predictor and corresponds to the logarithm of the (conditional) mean sales price or rent of apartment i, i.e. $\eta_i = \log(\mu_i)$; D_i is a dummy-variable with a value equal to one when the apartment i was rented and equal to zero when it was sold; Q_i includes the time information, i.e. in which (cumulative) quarter[5] the apartment i was sold or rented; YOC_i is the year of construction; $AREA_i$ is the living area; $LONG_i$ and LAT_i is the exact location, i.e. longitude and latitude; Y_{ki} is a set of dummy-variables indicating the year when the unit i was sold or rented[6]; the columns of the matrix X_i include all the other structural and locational attributes described in Table 1 which were not used so far. The functions f_1 to f_8 are all unspecified and have to be estimated adequately.[7]

Note that we do not include time in the nonparametric part where we model the effect of the exact location of the apartment (as proposed by Clapp (2004)) but allow for a yearly update of the surface on longitudes and latitudes. The reasons are: (i) We believe that the corresponding relationship is more stable over time and does not change, for example, each quarter. (ii) We reduce the dimensionality of the problem this way imposing more structure (here additivity) in the statistical estimation procedure, as proposed in the statistical literature by Stone (1985). This allows the application of the model also to smaller data sets, as, for example, for the city of Karlsruhe (see Table 2).

Note further, that the model in (3) is not designed for a ceteris-paribus analysis of the estimated shadow prices. Percentaged differences in fitted sales prices and rents caused by different effects of the characteristic on them are captured by the sales- and rent-specific intercepts (but jointly for all of the attributes).

Moreover, we assume that rents and sales have joint functions f_2 to f_8 depending on the year of construction, the living area, or on the geographical position. The reason here is to avoid an over-fitting of the data and a simplification in the estimation of the quality-adjusted price-rent ratios. This is described next.

With help of our model in (3), we can write the linear predictor separately for sales prices and rents (but still under the same joint model):

$$\begin{aligned}\eta_i^{\text{rent}} &= \beta_0^{\text{rent}} + f_1^{\text{rent}}(Q_i) + f_2(YOC_i) + f_3(AREA_i)\\ &\quad + \sum_{k=1}^{5} f_{k+3}(LONG_i, LAT_i)Y_{ki} + \boldsymbol{X}_i\beta \qquad (4)\\ \eta_i^{\text{sale}} &= \beta_0^{\text{sale}} + f_1^{\text{sale}}(Q_i) + f_2(YOC_i) + f_3(AREA_i)\\ &\quad + \sum_{k=1}^{5} f_{k+3}(LONG_i, LAT_i)Y_{ki} + \boldsymbol{X}_i\beta \qquad (5)\end{aligned}$$

where $\eta_i^{\text{rent}} = \log(\mu_i^{\text{rent}})$ for the (conditional) mean of rents μ_i^{rent} and $\eta_i^{\text{sale}} = \log(\mu_i^{\text{sale}})$ for the (conditional) mean of sales prices μ_i^{sale}. Note that the ratio of the two means does not depend on unit-specific characteristics. Thus, this value gives us the quality-adjusted price-rent ratio which depends only on time Q:

$$\frac{\mu^{\text{sale}}}{\mu^{\text{rent}}} = \exp\left((\beta_0^{\text{sale}} - \beta_0^{\text{rent}}) + (f_1^{\text{sale}}(Q) - f_1^{\text{rent}}(Q))\right). \qquad (6)$$

Note that with our approach it is not possible to create unit- or market-specific price-rent ratios. For this purpose other models are better suited. For example, Hill and Syed (2016) compute quality-adjusted price-rent ratios by ordering the rented and sold dwellings each year from the cheapest to the most expensive. Afterwards they computed the price-rent ratios for the lower quartile, the median, and upper quartile, finding that the price-rent ratio increased from the lower to the upper end of the market. Nevertheless, our approach gives valuable information on the aggregate-mean view which is important, for example, for financial stability purposes of central banks, financial investors, or other market participants.

[5]Our data set covers the period 2014Q2 to 2018Q1. Thus we have a time span of 16 quarters.

[6]For example, $Y_{1i} = 1$ when the apartment i was sold/rented in the year 2014 and $Y_{1i} = 0$ otherwise.

[7]The advantage of this type of smooth modelling is, that, for example, we could produce results for any frequency of time without the need of interpolation.

Quality-adjusted property price and rental indices

As a side-product of our modeling process, we can directly derive quality-adjusted property price and rental indices. For this purpose, we extend the well-known Time-Dummy Method (TDM) which in its standard for can be formulated as (see, for example, Hill (2013))

$$y = Z\beta + D\delta + \varepsilon, \tag{7}$$

where y is a vector of log sales prices or rents, Z the matrix that captures the structural and locational characteristics, D a matrix of period dummies (for example, as in our case for the quarters of interest), and ε the error-term with the usual properties. The attraction of this formulation is that the price index P_t for period t is derived by exponentiating the corresponding element of the estimated coefficient δ_t, i.e. $\hat{P}_t = \exp\hat{\delta}_t$. Note that depending on the assumptions in the hedonic model, corrections in the back-transformation should be incorporated, see, for example, Goldberger (1968) or Wooldrige (2012).

When we now compare equation (7) with our linear predictors for sales prices and rents in (4) and (5), we observe a similar structure. The main difference is that the time trends for sales prices and rents in (4) and (5) are modeled with flexible functions f_1^{sale} and f_1^{rent} instead of time-dummies. Note further, that the mentioned caution in the back-transformation is not necessary since we have modeled the linear predictor, the log of the mean sales price or rent, and not the observed response directly. Thus, our estimates for quality-adjusted price and rental indices are as follows:

$$\hat{P}_t^{\text{sale}} = \exp\left(\hat{f}_1^{\text{sale}}(Q_t)\right) \quad \text{and} \quad \hat{P}_t^{\text{rent}} = \exp\left(\hat{f}_1^{\text{rent}}(Q_t)\right). \tag{8}$$

For a discussion on strengths and weaknesses of the TDM and a possible alternative ways in producing quality-adjusted price indices, see, for example, the survey of Hill (2013).

3 The user-cost approach and expected capital gains

In this section, we want to make use of our estimated quality-adjusted price-rent ratios for producing an estimate of expected capital gains. Here we closely follow the approach used by Hill and Syed (2016).

We start with the user-cost equilibrium condition, adopted to our housing context. In equilibrium, the user cost of an apartment, u_tP_t, which is represented by its present value of buying it, using it one period and selling it afterwards equals the cost of renting the apartment for one period, R_t, (see, for example, Hicks (1946) and Himmelberg et al. (2005)):

$$R_t = u_tP_t, \tag{9}$$

where R_t is the rental price in period t, P_t is the sales price in period t, and u_t is the user cost per monetary unit. Following Hill and Syed (2016), the user cost per monetary unit can be stated in the following way:

$$u_t = r_t + \omega_t + \delta_t + \gamma_t - g_t, \tag{10}$$

where r represents an interest rate, ω the running and average transaction costs, δ the depreciation rate for housing, γ the risk premium of owning as opposed to renting, and g the expected capital gains. Rearranging the user cost formula (10) and making use of the equilibrium condition (9), we can express the expected (nominal) capital gains[8] as:

$$g_t = r_t + \omega_t + \delta_t + \gamma_t - \frac{R_t}{P_t}. \tag{11}$$

[8] The choice of the different components which are necessary for the computation of the expected capital gains is discussed in Section 4.

Table 3: Quality-adjusted price-rent ratios for 21 major cities in Germany

City name	14Q2	14Q3	14Q4	15Q1	15Q2	15Q3	15Q4	16Q1	16Q2	16Q3	16Q4	17Q1	17Q2	17Q3	17Q4	18Q1
Berlin	24.08	24.01	24.34	24.76	25.15	25.50	25.47	25.19	25.63	26.56	27.13	27.40	27.70	28.13	28.39	28.47
	(0.066)	(0.055)	(0.040)	(0.042)	(0.061)	(0.046)	(0.043)	(0.052)	(0.049)	(0.056)	(0.059)	(0.062)	(0.057)	(0.055)	(0.050)	(0.073)
Cologne	22.27	22.41	22.60	22.86	23.13	23.40	23.65	23.92	24.21	24.53	24.87	25.23	25.61	25.99	26.28	26.48
	(0.100)	(0.067)	(0.060)	(0.063)	(0.065)	(0.065)	(0.063)	(0.065)	(0.071)	(0.076)	(0.085)	(0.085)	(0.084)	(0.078)	(0.079)	(0.114)
Düsseldorf	23.58	23.87	24.00	23.97	23.99	24.23	24.60	24.69	25.09	26.13	26.52	25.85	25.59	26.55	27.48	27.85
	(0.138)	(0.094)	(0.082)	(0.104)	(0.114)	(0.118)	(0.087)	(0.103)	(0.126)	(0.113)	(0.164)	(0.131)	(0.130)	(0.111)	(0.110)	(0.158)
Frankfurt	24.56	24.84	25.14	25.46	25.75	25.97	26.17	26.52	26.95	27.35	27.71	28.10	28.66	29.44	30.06	30.45
	(0.130)	(0.086)	(0.079)	(0.088)	(0.093)	(0.093)	(0.086)	(0.090)	(0.099)	(0.101)	(0.120)	(0.117)	(0.116)	(0.107)	(0.106)	(0.154)
Hamburg	25.71	26.26	26.51	26.65	27.24	27.70	27.68	27.76	28.26	29.10	29.66	29.73	29.86	30.38	30.68	30.59
	(0.122)	(0.083)	(0.075)	(0.090)	(0.098)	(0.101)	(0.084)	(0.089)	(0.106)	(0.101)	(0.140)	(0.115)	(0.113)	(0.097)	(0.094)	(0.130)
Munich	29.80	30.12	30.51	30.99	31.45	31.63	31.63	32.08	32.80	33.26	33.02	32.22	31.74	32.10	32.69	33.18
	(0.135)	(0.094)	(0.085)	(0.103)	(0.111)	(0.115)	(0.097)	(0.105)	(0.124)	(0.119)	(0.158)	(0.124)	(0.118)	(0.102)	(0.100)	(0.141)
Stuttgart	22.81	22.86	22.93	23.06	23.27	23.27	23.21	23.80	24.11	23.50	23.39	24.47	25.15	24.52	24.17	24.62
	(0.193)	(0.131)	(0.122)	(0.146)	(0.149)	(0.152)	(0.142)	(0.147)	(0.168)	(0.157)	(0.203)	(0.175)	(0.178)	(0.149)	(0.144)	(0.208)
Bochum	16.53	16.92	17.23	17.41	17.44	17.38	17.28	17.18	17.09	16.98	16.90	16.93	17.23	17.79	18.15	18.20
	(0.182)	(0.116)	(0.107)	(0.126)	(0.136)	(0.129)	(0.104)	(0.110)	(0.120)	(0.116)	(0.146)	(0.140)	(0.139)	(0.128)	(0.123)	(0.180)
Bonn	21.523	21.769	21.882	21.87	21.899	22.058	22.33	22.587	22.849	23.15	23.398	23.539	23.604	23.66	23.832	24.127
	0.166	0.107	0.097	0.107	0.113	0.11	0.099	0.106	0.115	0.113	0.132	0.133	0.133	0.12	0.115	0.171
Bremen	18.028	18.505	18.902	19.154	19.279	19.573	20.134	20.627	21.054	21.478	21.658	21.565	21.574	21.929	22.425	22.927
	0.184	0.118	0.108	0.124	0.138	0.14	0.126	0.14	0.156	0.148	0.185	0.175	0.168	0.142	0.133	0.194
Dortmund	17.361	17.611	17.56	17.282	17.201	17.326	17.411	17.162	17.269	18.105	18.317	17.461	16.95	17.549	18.147	18.268
	0.149	0.101	0.088	0.114	0.129	0.132	0.088	0.098	0.116	0.102	0.148	0.116	0.114	0.099	0.096	0.139
Dresden	20.746	20.51	20.349	20.339	20.534	20.833	21.039	20.966	20.926	21.19	21.442	21.511	21.595	21.867	22.329	22.914
	0.102	0.063	0.055	0.067	0.079	0.081	0.058	0.066	0.075	0.063	0.092	0.083	0.082	0.065	0.061	0.089
Duisburg	14.643	14.255	14.096	14.024	13.672	13.571	13.777	13.362	13.078	13.606	13.845	13.383	13.332	14.28	14.997	15.011
	0.157	0.097	0.081	0.103	0.111	0.112	0.075	0.079	0.088	0.081	0.114	0.087	0.087	0.085	0.088	0.129
Essen	16.438	16.216	16.033	15.892	15.786	15.836	16.044	16.189	16.216	16.182	16.185	16.324	16.697	17.276	17.645	17.717
	0.126	0.078	0.069	0.081	0.089	0.086	0.071	0.078	0.087	0.08	0.104	0.093	0.092	0.084	0.081	0.12
Hanover	19.32	19.684	20.009	20.289	20.555	20.804	21.07	21.441	21.852	22.225	22.641	23.134	23.628	24.012	24.117	23.986
	0.148	0.095	0.087	0.095	0.099	0.098	0.092	0.095	0.103	0.107	0.125	0.12	0.119	0.11	0.104	0.15
Karlsruhe	22.053	22.258	22.45	22.573	22.558	22.532	22.698	23.224	23.895	24.417	24.778	25.035	25.289	25.611	25.958	26.302
	0.218	0.143	0.13	0.14	0.148	0.148	0.138	0.146	0.163	0.171	0.203	0.201	0.195	0.175	0.168	0.236
Leipzig	18.436	18.605	18.627	18.499	18.441	18.956	19.818	19.67	19.645	20.899	21.907	21.751	21.423	21.701	22.302	22.977
	0.114	0.081	0.068	0.086	0.091	0.094	0.071	0.08	0.097	0.08	0.124	0.097	0.096	0.08	0.079	0.108
Mannheim	20.188	20.413	20.384	20.182	20.224	20.647	21.228	21.424	21.643	22.305	22.769	22.74	22.742	23.131	23.553	23.848
	0.205	0.132	0.116	0.132	0.14	0.142	0.121	0.131	0.149	0.14	0.18	0.156	0.151	0.13	0.13	0.184
Münster	22.556	23.793	24.107	23.708	23.936	24.492	24.745	24.587	24.628	25.268	25.774	25.767	25.716	26.057	26.902	28.126
	0.265	0.173	0.155	0.184	0.204	0.209	0.169	0.187	0.216	0.198	0.27	0.238	0.236	0.208	0.205	0.306
Nuremberg	21.287	21.838	22.172	22.398	22.845	22.751	22.137	22.488	22.912	22.396	22.277	23.098	23.746	23.497	23.416	23.866
	0.195	0.134	0.126	0.147	0.154	0.156	0.135	0.147	0.173	0.161	0.213	0.184	0.185	0.153	0.146	0.201
Wiesbaden	21.888	22.521	22.967	23.267	23.667	23.9	23.901	24.151	24.529	24.794	25.149	25.622	25.859	25.725	25.818	26.284
	0.203	0.136	0.126	0.147	0.162	0.164	0.133	0.143	0.162	0.155	0.209	0.191	0.187	0.159	0.149	0.212

Notes: (i) The table reports estimated price-rent ratios and in brackets their standard deviations for 21 A- and B-cites in Germany. (ii) For this purpose, a GAM displayed in (3) was estimated jointly on sales prices and rents. Finally, predictions of the mean for sales and rents are combined via equation (6).

Note that the equilibrium condition (9) implicitly assumes that prices and rents are calculated for apartments of the same quality.[9] But we have already taken this into account when we produced estimates of the quality-adjusted price-rent ratio (which enters reciprocally in equation (11)). Note further, that g could be separated into two components: the expected real capital gain and expected inflation. Usually it is assumed that with the access to long time-series expected real capital gains can be extrapolated from past performance of the real-estate market. Such series are hardly available for Germany, and thus we follow the described way for getting an estimate of expected capital gains.

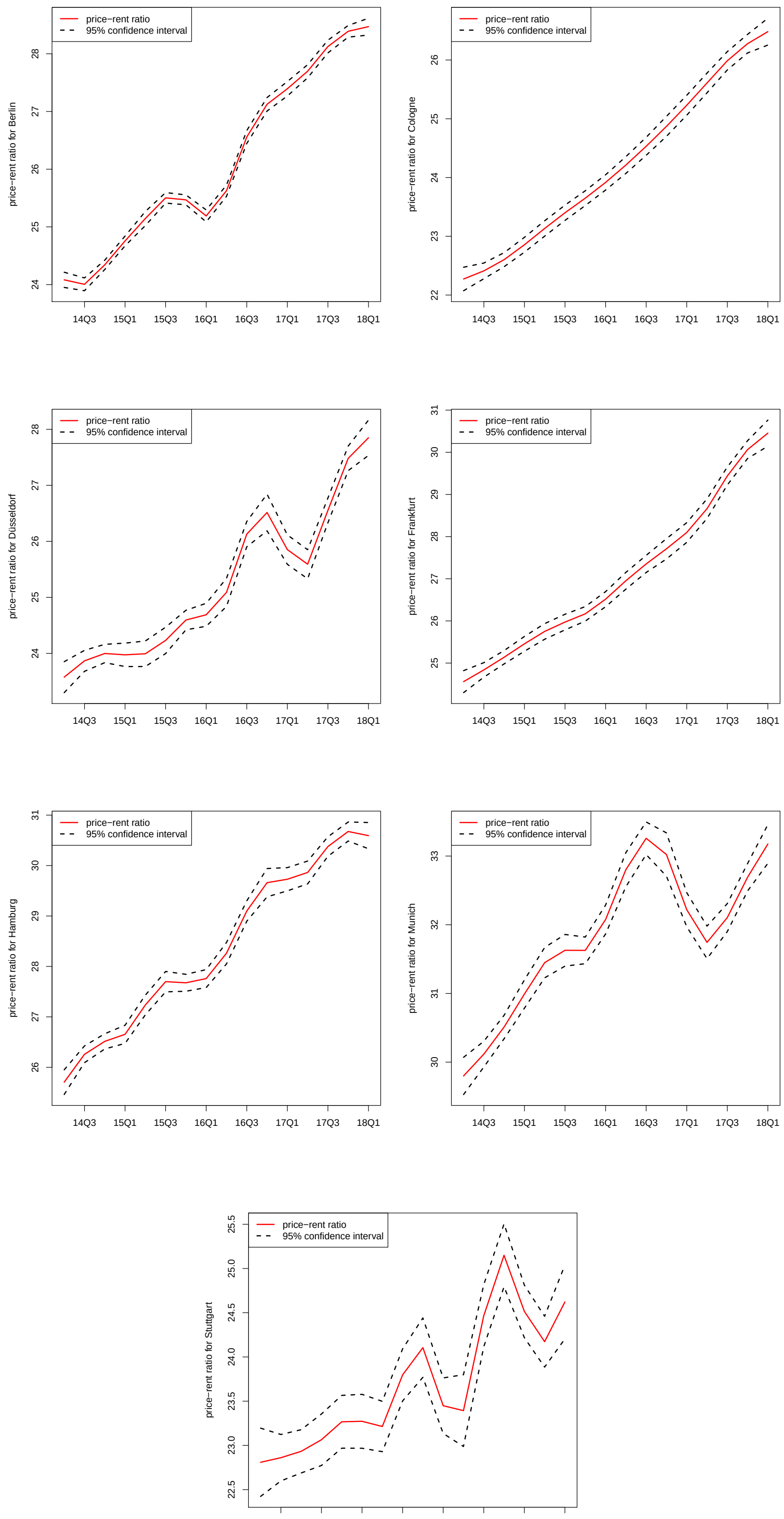
4 Empirical results

Quality-adjusted price-rent ratios

We apply the method described in Section 2 for each individual A- or B-city separately. Our results for the quality-adjusted price-rent ratios are shown in Table 3, and graphically for A-cities in Figure 1, and for B-cities in Figures 2–3.

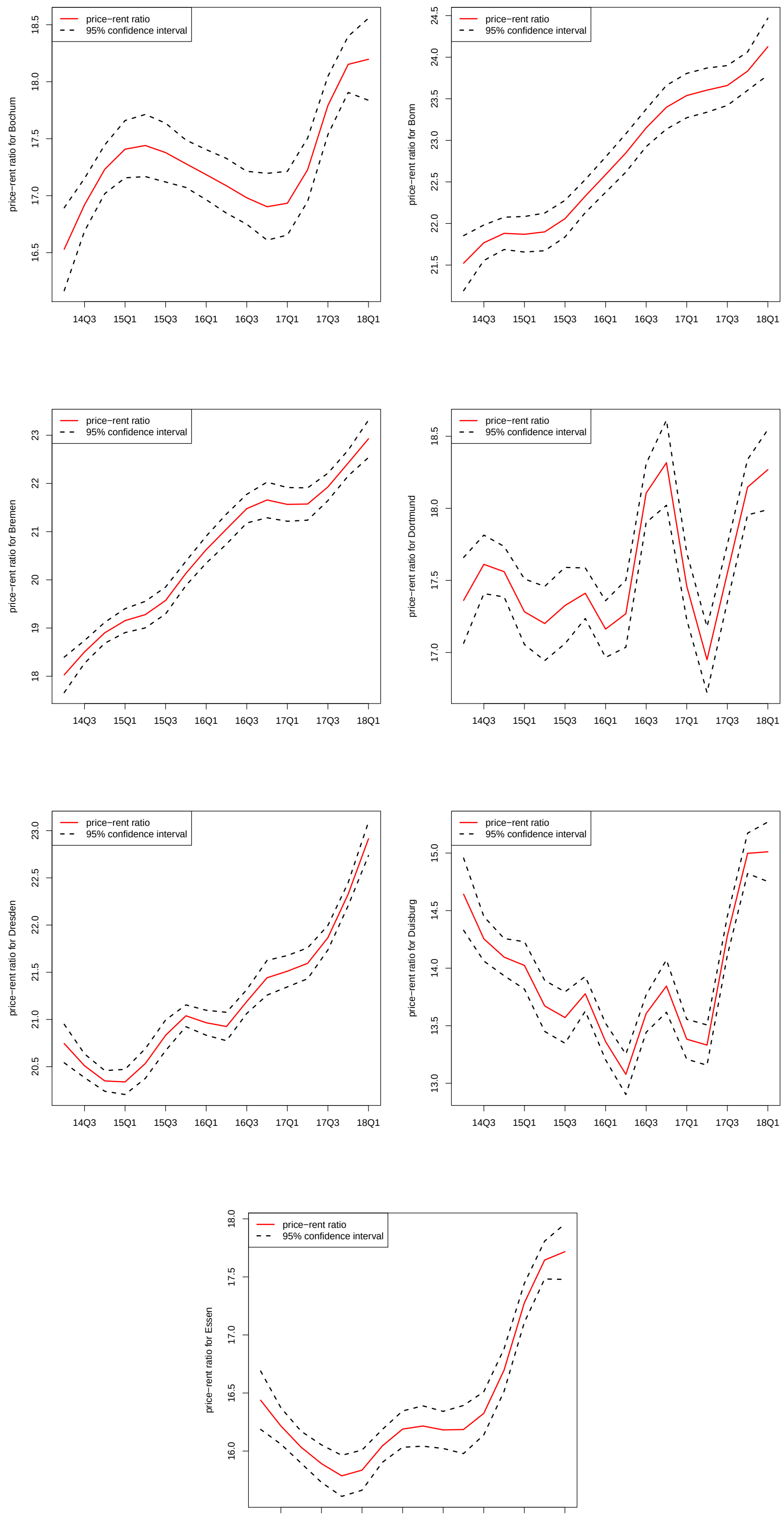
[9]Table 1 confirms the quality differences in apartments sold vs. rented. For example, on average sold apartments have more living space, are newer, and are better equipped with balcony, garden, or second bathroom.

Figure 1: Price-rent ratios for A-cities over the period 2014Q2 to 2018Q1.



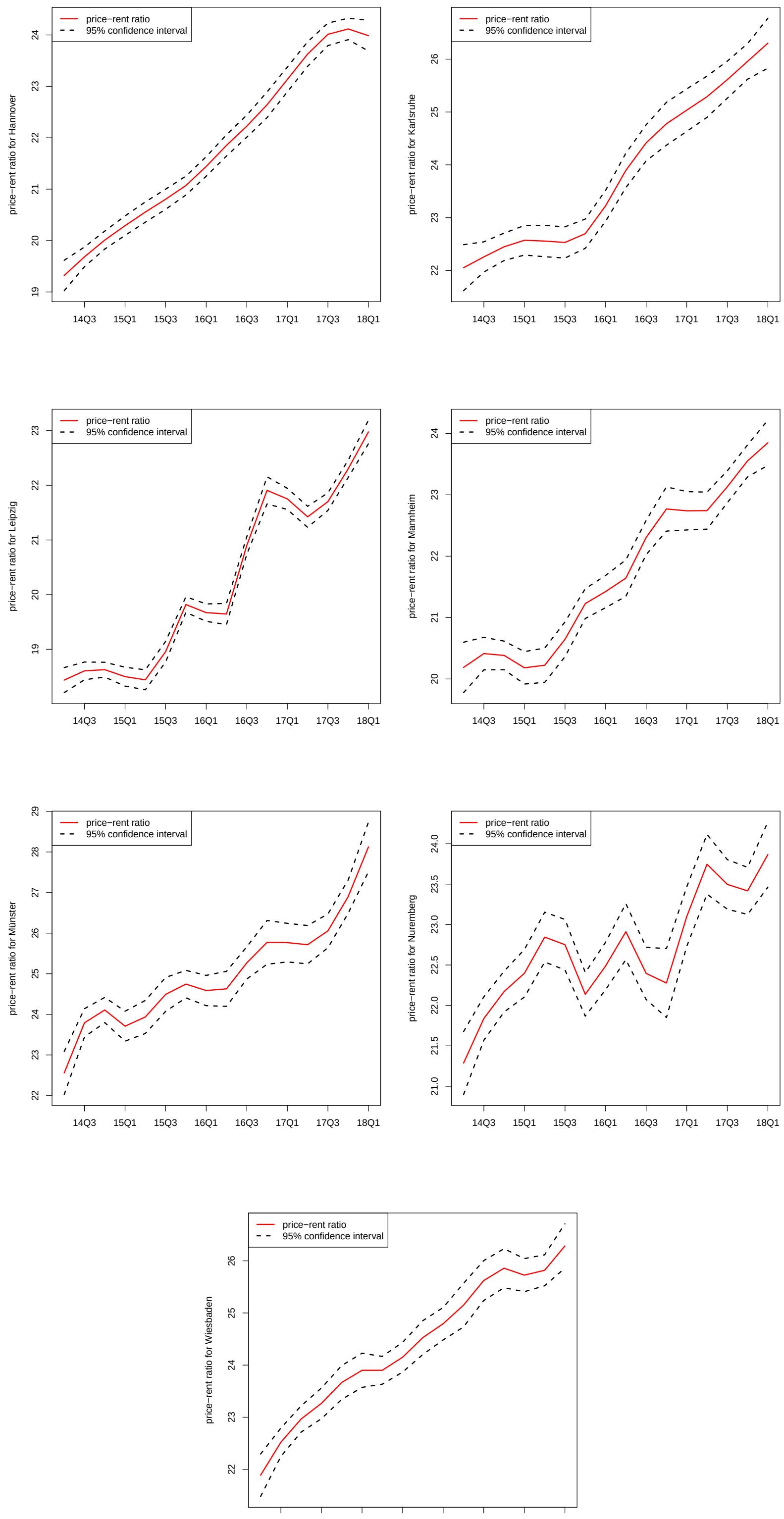
Notes: (i) The figure displays estimated quality-adjusted price-rent ratios (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for A-cities are shown: Berlin, Cologne, Düsseldorf, Frankfurt, Hamburg, Munich, and Stuttgart (linewise beginning at the top left panel)

Figure 2: Price-rent ratios for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays estimated quality-adjusted price-rent ratios (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Bochum, Bonn, Bremen, Dortmund, Dresden, Duisburg, and Essen (linewise beginning at the top left panel)

Figure 3: Price-rent ratios for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays estimated quality-adjusted price-rent ratios (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Hanover, Karlsruhe, Leipzig, Mannheim, Münster, Nuremberg, and Wiesbaden (linewise beginning at the top left panel)

In absolute terms, we find that Munich has the highest price rent ratio (in 2018Q1, the end of our sample period) with 33.2, followed by Hamburg (30.6), and Frankfurt (30.5). Practitioners value price-rent ratios larger or equal to 21 as high. Most of the cities, namely also Berlin (28.5), Münster (28.1), Düsseldorf (27.8), Cologne (26.5), Karlsruhe (26.3), Wiesbaden (26.3), Stuttgart (24.6), Bonn (24.1), Hanover (24.0), Nuremberg (23.9), Mannheim (23.8), Leipzig (23.0), Bremen (22.9), and Dresden (22.9) fulfil this criterion. A moderate price-rent ratio can be observed for Dortmund (18.3), Bochum (18.2), and Essen (17.7). Only the city of Duisburg (15.0) shows a low price-rent ratio.

When we consider the change of the price-rent ratios individually for each city, we get an overview of the cross-sectional dynamics for the German housing market. Calculating the relative change of the price-rent ratio over the whole sample period, we find a diversified development. The largest change in the price-rent ratio can be observed for Bremen with an increase of 27.2 percent, followed by Münster (24.7%), Leipzig (24.6%), Hanover (24.1%), Frankfurt (24.0%), and Wiesbaden (20.1%). Usually, prices increase first and rents catch up later, which explains this behavior to some extend.[10] We will come back to this point when we consider quality-adjusted residential sales price and rental indices. Interestingly, most of the cities with an increase of the price-rent ratio over 20 percent are B-cities, indicating that in those cities capital gains could be (or have been) higher for investors. Moderate changes in the price-rent ratio are found for Karlsruhe (19.3%), Hamburg (19.0%), Cologne (18.9%), Berlin (18.2%), Mannheim (18.1%), Düsseldorf (18.1%), Nuremberg (12.1%), Bonn (12.1%), Munich (11.3%), Dresden (10.5%), and Bochum (10.1%). In contrast, only small changes are observed for Stuttgart (7.9%), Essen (7.8%), Dortmund (5.2%), and Duisburg (2.5%).

Figures 1–3 also show that some cities like Cologne or Frankfurt experienced a continuous growth of the price-rent ratio over the whole sample period while other cities like Munich or Düsseldorf had a slide downturn around 2017Q2. Note that most cities had a sharp increase of the price-rent ratio from 2017Q2 to 2018Q1 in common.

Quality-adjusted property price and rental indices

As described in Section 2, we can use the estimated time-trends for sales prices and rents, $\hat{f}_1^{\text{sale}}$ and $\hat{f}_1^{\text{rent}}$, to produce quality-adjusted price and rental indices again at the city-level. Results for sales prices are presented in Table 4 and for rents in Table 5. Figure 4 shows for A-cities the development over time in both rental and sales market together. For B-cities Figures 5–6 present the corresponding graphs. Note that we normalize all indices to one in 2014Q2.

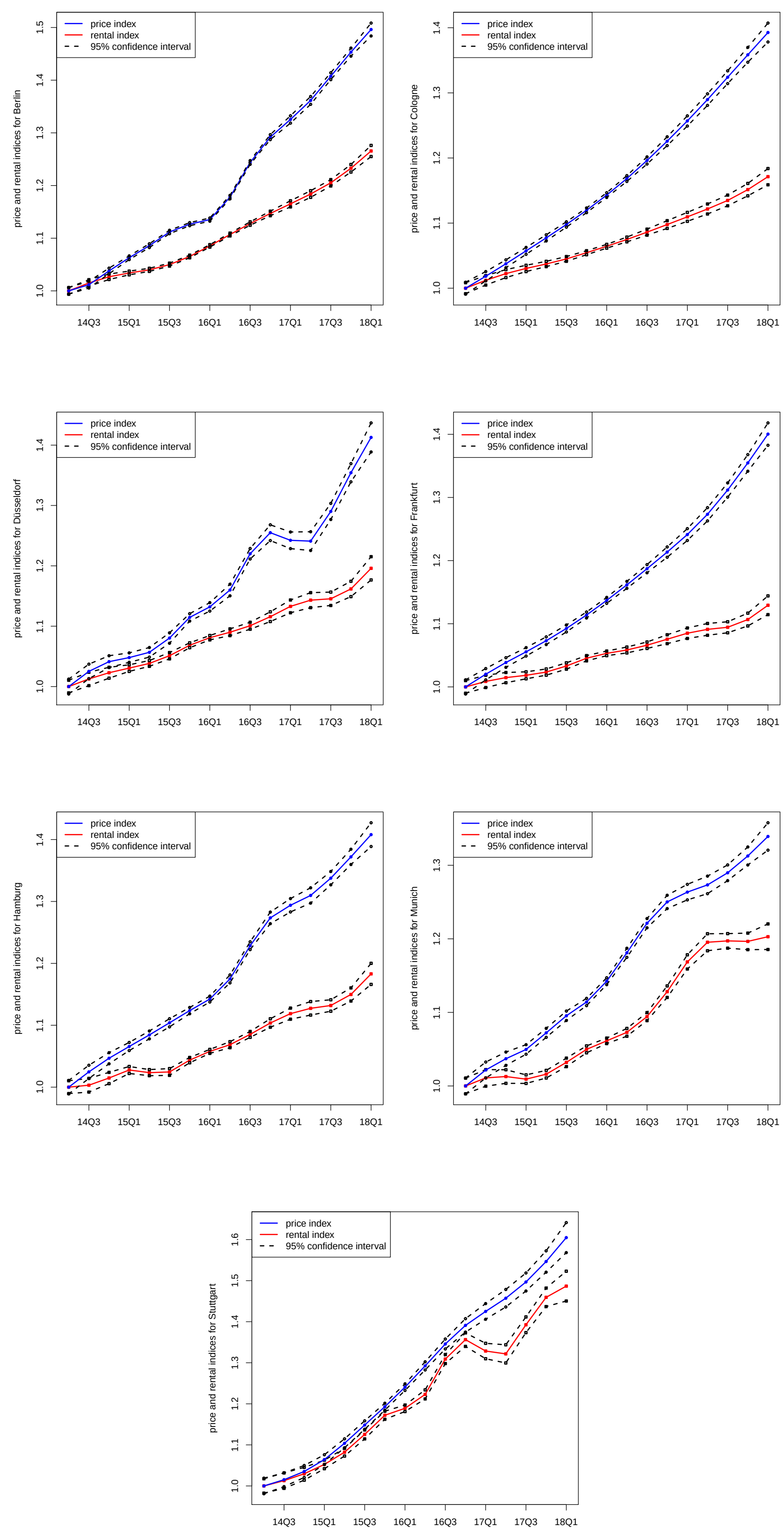
The largest increase in sales prices over the sample period can be observed for Stuttgart with 60.5%, followed by Leipzig (57.3%), Nuremberg (56.9%), Karlsruhe (50.5%), Mannheim (50.1%), Berlin (49.6%), and Bremen (47.7%). All of the mentioned cities experienced an average compound annual growth between 12.6% and 10.2%. For Hanover (46.3%), Wiesbaden (44.0%), Münster (42.6%), Düsseldorf (41.3%), Hamburg (40.8%), Frankfurt (40.0%), Cologne (39.3%), Munich (33.9%), Dortmund (30.6%), Essen (28.7%), Dresden (27.0%), Bochum (26.7%), and Bonn (24.7%) we find an average compound annual growth between 9.9% and 5.7%. The apartment sales price for Duisburg (13.5%) grew annually on average with the smallest rate of 3.2%.

As expected, the increase in rents is smaller compared to sales prices. The largest increase is again found for Stuttgart with 48.7%, followed by Nuremberg (39.9%), Mannheim (27.1%), Berlin (26.6%), Leipzig (26.2%), Karlsruhe (26.2%), and Dortmund (24.1%). This correspondes to an average compound annual growth rate for apartment rents between 10.4% and 5.5%. For Munich (20.3%), Wiesbaden (19.9%), Düsseldorf (19.6%), Essen (19.4%), Hamburg (18.3%), Hanover (17.8%), Cologne (17.1%), Bremen (16.2%), Bochum (15.1%), Dresden (15.0%), Münster (14.4%), Frankfurt (12.9%), Bonn (11.2%), and Duisburg (10.7%). These cities experienced a moderate average compound annual growth rate for apartment rents between 4.7% and 2.6%.

Comparing the increases over the sample period in both markets at the same time, we find the largest gap in the development of sales prices vs. rents for Bremen with a difference of 31.5 percentage points, followed by Leipzig (31.1), Hanover (28.5), Münster (28.2), and Frankfurt (27.1). Note that those cities also had the largest changes in the price-rent ratios, clearly driven by the faster growth of apartment sales prices. But not for all cities we observe this large

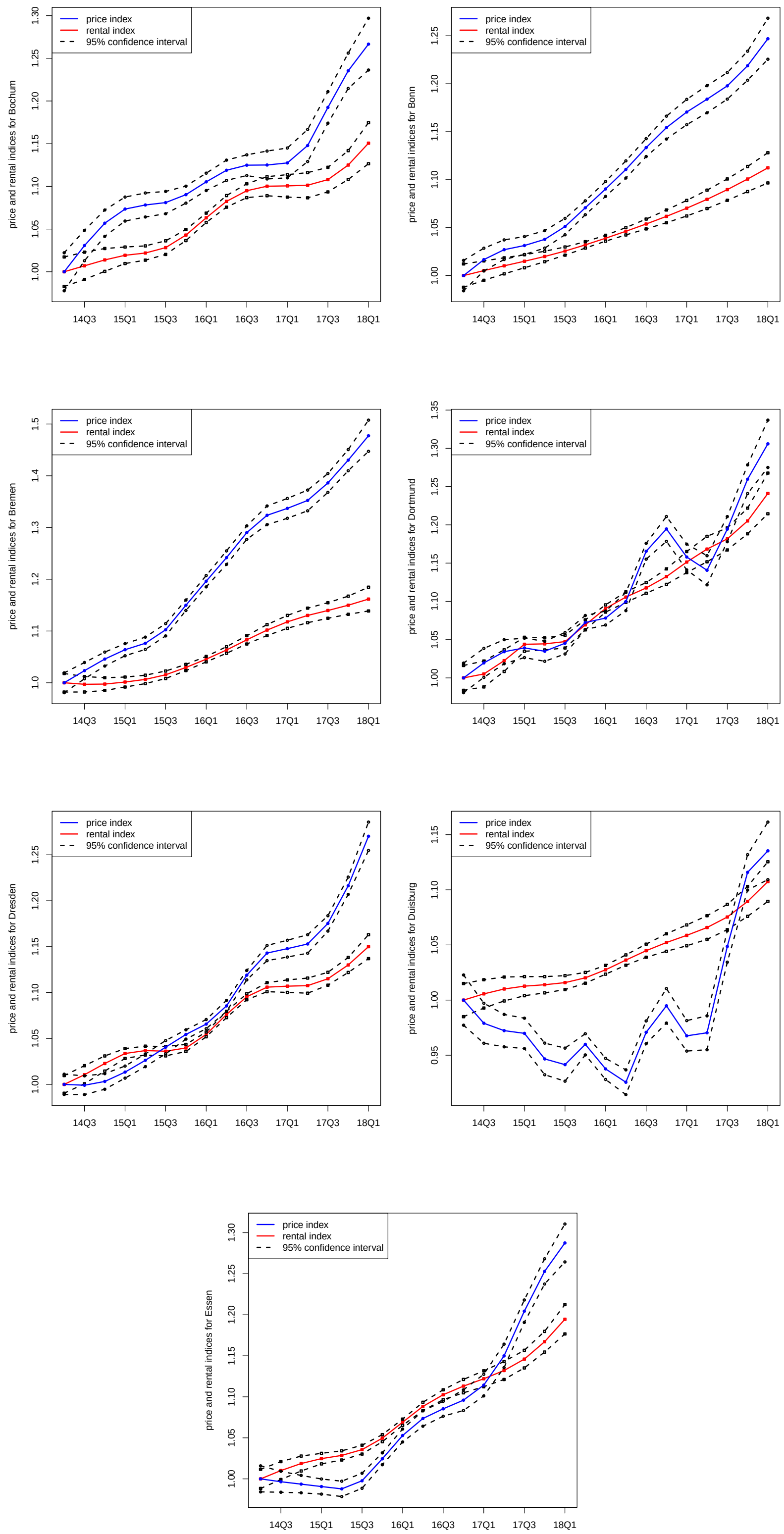
[10]Under the assumption of constant rents over the sample period, the increase in the price-rent ratio of 27.2% for Bremen would translate to a lower bound of the average compound annual growth rate of 6.2% in the value of apartments.

Figure 4: Price and rental indices for A-cities over the period 2014Q2 to 2018Q1.



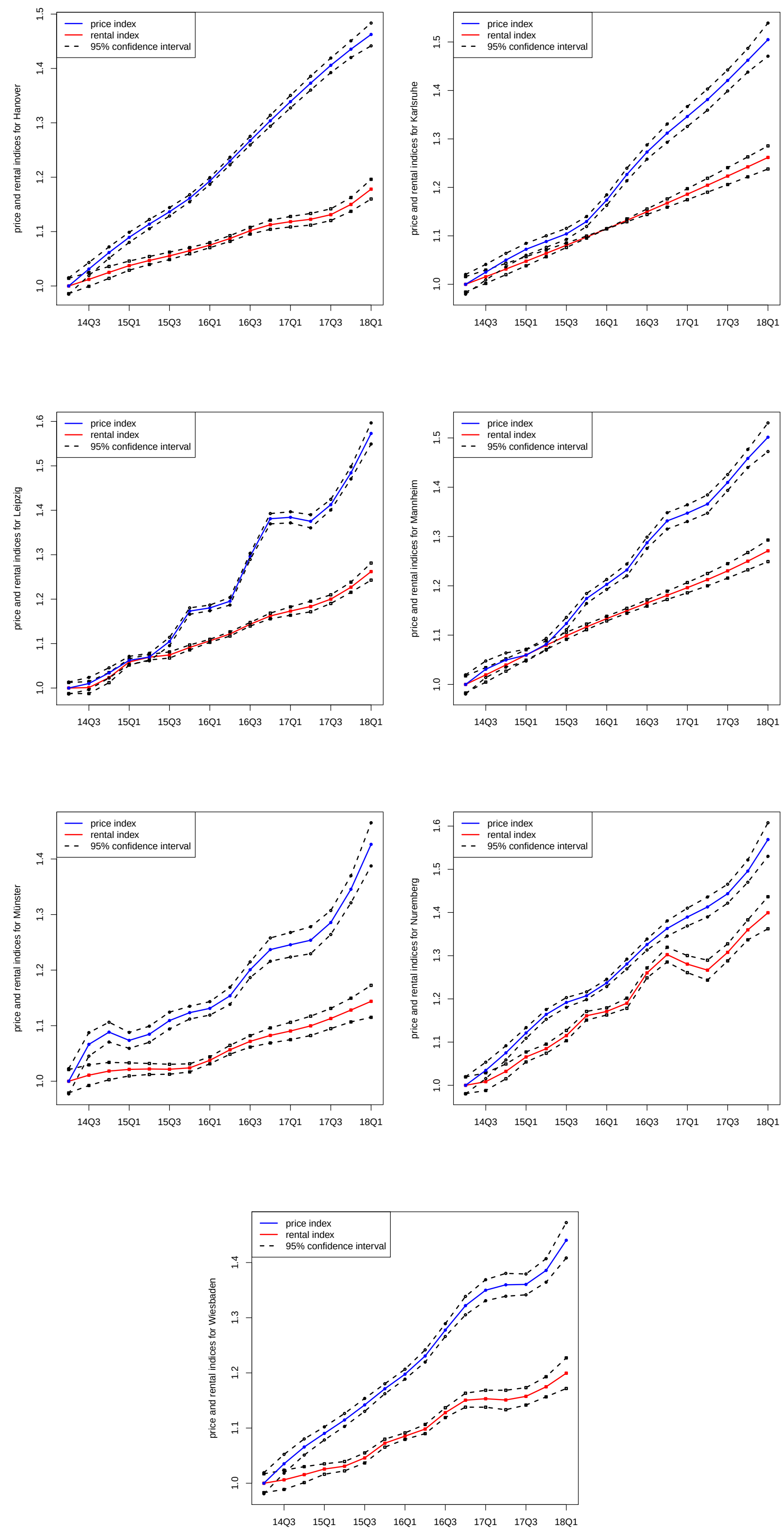
Notes: (i) The figure displays estimated quality-adjusted price indices (blue solid) and rental indices (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for A-cities are shown: Berlin, Cologne, Düsseldorf, Frankfurt, Hamburg, Munich, and Stuttgart (linewise beginning at the top left panel)

Figure 5: Price and rental indices for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays estimated quality-adjusted price indices (blue solid) and rental indices (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Bochum, Bonn, Bremen, Dortmund, Dresden, Duisburg, and Essen (linewise beginning at the top left panel)

Figure 6: Price and rental indices for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays estimated quality-adjusted price indices (blue solid) and rental indices (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Hanover, Karlsruhe, Leipzig, Mannheim, Münster, Nuremberg, and Wiesbaden (linewise beginning at the top left panel)

Table 4: Quality-adjusted sales price indices for 21 major cities in Germany

City name	14Q2	14Q3	14Q4	15Q1	15Q2	15Q3	15Q4	16Q1	16Q2	16Q3	16Q4	17Q1	17Q2	17Q3	17Q4	18Q1
Berlin	1.000	1.012	1.038	1.063	1.086	1.112	1.127	1.135	1.178	1.244	1.292	1.325	1.362	1.407	1.453	1.496
	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.003)	(0.004)	(0.006)
Cologne	1.000	1.018	1.038	1.057	1.077	1.098	1.120	1.143	1.168	1.196	1.226	1.257	1.290	1.324	1.358	1.393
	(0.005)	(0.004)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.006)	(0.007)
Düsseldorf	1.000	1.025	1.041	1.048	1.057	1.080	1.115	1.132	1.160	1.220	1.255	1.242	1.241	1.290	1.354	1.413
	(0.006)	(0.006)	(0.005)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.005)	(0.004)	(0.007)	(0.007)	(0.008)	(0.007)	(0.008)	(0.012)
Frankfurt	1.000	1.020	1.039	1.056	1.073	1.093	1.114	1.137	1.162	1.187	1.213	1.241	1.273	1.312	1.355	1.400
	(0.006)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)	(0.005)	(0.006)	(0.007)	(0.009)
Hamburg	1.000	1.025	1.047	1.066	1.084	1.104	1.124	1.142	1.175	1.229	1.273	1.294	1.310	1.338	1.372	1.408
	(0.005)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.010)
Munich	1.000	1.022	1.037	1.050	1.072	1.095	1.114	1.142	1.181	1.221	1.250	1.263	1.273	1.290	1.313	1.339
	(0.005)	(0.005)	(0.005)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)	(0.009)
Stuttgart	1.000	1.015	1.035	1.064	1.104	1.148	1.193	1.241	1.292	1.346	1.391	1.425	1.457	1.497	1.547	1.605
	(0.009)	(0.008)	(0.007)	(0.006)	(0.005)	(0.005)	(0.004)	(0.004)	(0.005)	(0.006)	(0.008)	(0.010)	(0.011)	(0.011)	(0.013)	(0.010)
Bochum	1	1.031	1.057	1.073	1.078	1.081	1.09	1.105	1.119	1.125	1.125	1.127	1.148	1.192	1.235	1.267
	0.011	0.009	0.008	0.007	0.007	0.007	0.005	0.005	0.006	0.006	0.008	0.009	0.009	0.009	0.01	0.015
Bonn	1	1.017	1.027	1.031	1.038	1.051	1.071	1.09	1.111	1.133	1.154	1.17	1.184	1.198	1.219	1.247
	0.008	0.006	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.005	0.006	0.007	0.007	0.007	0.008	0.011
Bremen	1	1.023	1.046	1.064	1.076	1.102	1.15	1.196	1.242	1.29	1.324	1.337	1.352	1.386	1.43	1.477
	0.01	0.008	0.007	0.006	0.006	0.006	0.005	0.005	0.007	0.007	0.009	0.01	0.01	0.009	0.01	0.015
Dortmund	1	1.02	1.034	1.039	1.035	1.045	1.072	1.078	1.1	1.165	1.195	1.158	1.141	1.194	1.26	1.306
	0.01	0.009	0.008	0.006	0.007	0.007	0.005	0.005	0.006	0.005	0.008	0.008	0.01	0.008	0.009	0.015
Dresden	1	0.999	1.003	1.013	1.026	1.041	1.054	1.066	1.085	1.119	1.143	1.148	1.153	1.175	1.216	1.27
	0.006	0.005	0.004	0.003	0.003	0.004	0.003	0.002	0.003	0.003	0.004	0.005	0.005	0.004	0.005	0.008
Duisburg	1	0.979	0.972	0.97	0.947	0.941	0.96	0.938	0.926	0.971	0.995	0.968	0.97	1.049	1.116	1.135
	0.011	0.009	0.007	0.007	0.007	0.008	0.005	0.005	0.006	0.005	0.008	0.007	0.008	0.007	0.008	0.013
Essen	1	0.997	0.994	0.991	0.988	0.998	1.024	1.053	1.074	1.085	1.096	1.114	1.15	1.204	1.253	1.287
	0.008	0.006	0.005	0.005	0.005	0.005	0.004	0.004	0.005	0.005	0.006	0.007	0.007	0.007	0.008	0.012
Hanover	1	1.031	1.062	1.089	1.114	1.136	1.161	1.193	1.23	1.267	1.304	1.339	1.373	1.406	1.435	1.463
	0.008	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.004	0.005	0.006	0.006	0.007	0.008	0.01
Karlsruhe	1	1.025	1.05	1.072	1.088	1.104	1.13	1.174	1.227	1.273	1.312	1.346	1.381	1.421	1.462	1.505
	0.01	0.008	0.007	0.006	0.006	0.006	0.005	0.005	0.006	0.007	0.009	0.01	0.011	0.011	0.012	0.017
Leipzig	1	1.01	1.034	1.062	1.07	1.105	1.173	1.18	1.195	1.296	1.381	1.384	1.375	1.413	1.484	1.573
	0.007	0.007	0.006	0.004	0.004	0.005	0.004	0.003	0.004	0.003	0.006	0.006	0.007	0.006	0.007	0.012
Mannheim	1	1.031	1.049	1.059	1.081	1.124	1.174	1.203	1.232	1.287	1.332	1.347	1.366	1.41	1.458	1.501
	0.01	0.008	0.007	0.006	0.006	0.006	0.005	0.005	0.006	0.006	0.008	0.008	0.009	0.008	0.009	0.015
Münster	1	1.066	1.088	1.074	1.085	1.109	1.123	1.131	1.154	1.201	1.237	1.246	1.254	1.286	1.345	1.426
	0.011	0.011	0.009	0.007	0.007	0.008	0.006	0.006	0.008	0.007	0.01	0.011	0.012	0.011	0.012	0.019
Nuremberg	1	1.035	1.075	1.121	1.164	1.192	1.207	1.237	1.281	1.326	1.363	1.389	1.413	1.444	1.496	1.569
	0.01	0.009	0.008	0.006	0.006	0.006	0.004	0.004	0.006	0.006	0.009	0.01	0.012	0.011	0.013	0.019
Wiesbaden	1	1.035	1.066	1.09	1.115	1.142	1.171	1.198	1.231	1.278	1.322	1.35	1.36	1.36	1.386	1.44
	0.01	0.009	0.007	0.006	0.006	0.006	0.005	0.005	0.006	0.006	0.008	0.01	0.01	0.009	0.011	0.016

Notes: (i) The table reports estimated sales price indices and in brackets their standard deviations for 21 A- and B-cites in Germany. (ii) For this purpose, a GAM displayed in (3) was estimated jointly on sales prices and rents. Finally, predictions for the smooth function f_1^{sale} are evaluated at the corresponding quarters. (iii) Indices are normalized to one in 2014Q2.

wedge between sales prices and rents. For example, in Stuttgart prices and rents grew at the same rate until 2016Q4. Then rents leveled off for two quarters and catched up at an even higher rate. For other cities like Dortmund, Dresden, Karlsruhe, Leipzig, or Mannheim we find a similar growth pattern for sales prices and rents until 2015Q3-2016Q2, starting afterwards to diverge. Clear exceptions are Duisburg and Essen, where prices even decreased, and rents grew faster than prices for most of the sample period. This is not an surprise as both cities lie in an area of an intensive structural change. But for both, starting with 2017Q3 this behavior is reversed and prices grow faster than rents.

Expected capital gains

Before we present the results of the estimation of real capital gains in a cross-section for the 21 German cities, we first discuss the choice of the different elements of (11): (i) For the nominal interest rate r_t, we follow Himmelberg et al. (2005) and use the 10-year Treasury interest rate.[11] [12] The German bond rate had a minimum of -0.12% in 2016Q3 and a maximum of 1.35% in 2014Q2. (ii) Average transaction costs are quite high in Germany compared

[11] Downloaded from `https://fred.stlouisfed.org/series/IRLTLT01DEQ156N`.

[12] An alternative could be a combination of the risk-free rate and the mortgage interest rate as discussed in Hill and Syed (2016). But to keep the analysis as simple as possible, we stick to the 10-year Treasury rate.

Table 5: Quality-adjusted rental indices for 21 major cities in Germany

City name	14Q2	14Q3	14Q4	15Q1	15Q2	15Q3	15Q4	16Q1	16Q2	16Q3	16Q4	17Q1	17Q2	17Q3	17Q4	18Q1
Berlin	1.000	1.015	1.027	1.034	1.040	1.050	1.065	1.085	1.107	1.128	1.147	1.165	1.184	1.205	1.233	1.266
	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.004)	(0.005)
Cologne	1.000	1.012	1.022	1.030	1.037	1.045	1.054	1.064	1.075	1.086	1.098	1.110	1.122	1.135	1.151	1.171
	(0.004)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.006)
Düsseldorf	1.000	1.013	1.023	1.031	1.038	1.051	1.068	1.081	1.090	1.101	1.116	1.133	1.143	1.145	1.162	1.196
	(0.005)	(0.006)	(0.005)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)	(0.006)	(0.006)	(0.006)	(0.010)
Frankfurt	1.000	1.009	1.015	1.018	1.024	1.033	1.045	1.053	1.058	1.066	1.076	1.085	1.091	1.094	1.107	1.129
	(0.005)	(0.005)	(0.004)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.004)	(0.005)	(0.004)	(0.005)	(0.007)
Hamburg	1.000	1.003	1.015	1.028	1.023	1.025	1.044	1.058	1.069	1.085	1.104	1.119	1.127	1.132	1.150	1.183
	(0.005)	(0.006)	(0.005)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)	(0.005)	(0.005)	(0.005)	(0.009)
Munich	1.000	1.011	1.013	1.009	1.016	1.032	1.050	1.061	1.073	1.094	1.128	1.169	1.195	1.197	1.197	1.203
	(0.005)	(0.006)	(0.005)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)	(0.006)	(0.005)	(0.006)	(0.009)
Stuttgart	1.000	1.013	1.030	1.052	1.082	1.126	1.172	1.189	1.223	1.309	1.356	1.329	1.322	1.393	1.459	1.487
	(0.009)	(0.009)	(0.008)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.006)	(0.005)	(0.008)	(0.009)	(0.011)	(0.010)	(0.011)	(0.018)
Bochum	1	1.007	1.014	1.019	1.022	1.028	1.043	1.063	1.082	1.095	1.1	1.101	1.101	1.108	1.125	1.151
	0.009	0.008	0.007	0.005	0.004	0.004	0.003	0.003	0.003	0.004	0.006	0.007	0.007	0.007	0.008	0.012
Bonn	1	1.005	1.01	1.015	1.02	1.026	1.032	1.039	1.046	1.054	1.062	1.07	1.079	1.09	1.101	1.112
	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006	0.007	0.008
Bremen	1	0.997	0.997	1.001	1.006	1.015	1.03	1.046	1.063	1.083	1.102	1.118	1.13	1.14	1.15	1.162
	0.009	0.007	0.006	0.005	0.004	0.004	0.003	0.003	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.011
Dortmund	1	1.005	1.023	1.044	1.044	1.047	1.069	1.091	1.106	1.118	1.132	1.151	1.168	1.182	1.205	1.241
	0.008	0.008	0.007	0.005	0.004	0.004	0.003	0.002	0.003	0.003	0.005	0.007	0.008	0.007	0.008	0.013
Dresden	1	1.011	1.023	1.034	1.037	1.036	1.04	1.054	1.076	1.095	1.106	1.107	1.108	1.115	1.13	1.15
	0.005	0.005	0.004	0.003	0.002	0.003	0.002	0.001	0.002	0.002	0.002	0.003	0.004	0.004	0.004	0.007
Duisburg	1	1.006	1.01	1.013	1.014	1.016	1.02	1.027	1.036	1.045	1.052	1.059	1.066	1.075	1.089	1.107
	0.008	0.006	0.005	0.004	0.004	0.003	0.002	0.002	0.002	0.003	0.004	0.005	0.005	0.006	0.007	0.009
Essen	1	1.01	1.019	1.025	1.029	1.036	1.05	1.069	1.088	1.103	1.113	1.122	1.132	1.146	1.167	1.194
	0.006	0.005	0.005	0.003	0.003	0.003	0.002	0.002	0.003	0.003	0.004	0.005	0.005	0.005	0.006	0.009
Hanover	1	1.012	1.025	1.037	1.047	1.055	1.065	1.075	1.087	1.102	1.113	1.118	1.123	1.131	1.15	1.178
	0.007	0.006	0.005	0.004	0.004	0.003	0.003	0.002	0.003	0.003	0.004	0.005	0.005	0.005	0.006	0.009
Karlsruhe	1	1.016	1.031	1.048	1.064	1.081	1.097	1.115	1.132	1.15	1.168	1.186	1.204	1.223	1.242	1.262
	0.008	0.007	0.006	0.005	0.004	0.002	0.001	0.001	0.001	0.003	0.004	0.006	0.007	0.009	0.010	0.012
Leipzig	1	1.001	1.023	1.059	1.069	1.074	1.091	1.106	1.122	1.144	1.162	1.173	1.183	1.200	1.227	1.262
	0.006	0.007	0.006	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.003	0.005	0.006	0.005	0.006	0.01
Mannheim	1	1.019	1.039	1.06	1.08	1.099	1.117	1.133	1.149	1.165	1.181	1.196	1.213	1.23	1.25	1.271
	0.009	0.007	0.006	0.005	0.004	0.004	0.003	0.002	0.003	0.003	0.004	0.005	0.006	0.007	0.009	0.011
Münster	1	1.011	1.018	1.021	1.022	1.022	1.024	1.038	1.057	1.072	1.082	1.09	1.1	1.113	1.128	1.144
	0.011	0.009	0.008	0.006	0.005	0.004	0.004	0.003	0.004	0.005	0.007	0.008	0.009	0.009	0.011	0.014
Nuremberg	1	1.008	1.032	1.066	1.085	1.115	1.161	1.171	1.19	1.26	1.302	1.281	1.267	1.308	1.36	1.399
	0.01	0.01	0.009	0.006	0.005	0.006	0.005	0.004	0.006	0.006	0.009	0.01	0.012	0.01	0.012	0.019
Wiesbaden	1	1.006	1.016	1.026	1.031	1.046	1.073	1.085	1.098	1.128	1.151	1.153	1.151	1.157	1.175	1.199
	0.009	0.009	0.007	0.005	0.004	0.005	0.004	0.003	0.004	0.004	0.006	0.008	0.009	0.008	0.009	0.014

Notes: (i) The table reports estimated rental indices and in brackets their standard deviations for 21 A- and B-cites in Germany. (ii) For this purpose, a GAM displayed in (3) was estimated jointly on sales prices and rents. Finally, predictions for the smooth function f_1^{rent} are evaluated at the corresponding quarters. (iii) Indices are normalized to one in 2014Q2.

to other countries.[13] They are composed of a transfer tax of 3.5%-6.5%[14] , notary fees of 0.8%-1.0%, and the entry in the land registry of 0.3%-0.5%, in total 4.6%-8.0% (see Voigtländer (2016)). For the running costs we follow Fox and Tulip (2016) and assume a value of 1.5%. This gives us in addition to the exact transfer tax ω_1 a value of $\omega_2 = 1.0\% + 0.5\% + 1.5 = 3.0\%$, which we consider constant over the sample period. (iii) The depriciation rate δ is fixed as well over the sample period and set to 2.5% (see Harding et al. (2007)). (iv) For the risk-premium we assume a constant $\gamma = 2.0\%$, as also used in Himmelberg et al. (2005). (v) Remeber that we are interested in real capital gains and have to account for expected inflation π_t^e. We use here the standard assumption in basic macro classes (see, for example, Blanchard (2017)), namely that expectations on inflation follow last periods inflation, i.e. $\pi_t^e = \pi_{t-1}$.[15]

[13] Voigtländer (2016) reports, for example, for the Netherlands 6.25%-6.50% or the UK 3.00%-3.25%, while Hill and Syed (2016) find 2% for Australia.

[14] We use the exact values: 3.5% (Bavaria, Saxony), 4.5% (Hamburg), 5.0% (Baden-Württemberg, Bremen, Mecklenburg-Western Pomerania, Lower Saxony, Rhineland-Palatinate, Saxony-Anhalt, Thuringia), 6.0% (Berlin, Hesse), 6.5% (Brandenburg, North Rine Westphalia, Saarland, Schleswig-Holstein).

[15] A simple alternative would be the assumption of $\pi_t^e = 2.0\%$, such that the inflation rate is ancored with the inflation target of the European Central Bank.

Table 6: Expected real capital gains for 21 major cities in Germany

City name	14Q2	14Q3	14Q4	15Q1	15Q2	15Q3	15Q4	16Q1	16Q2	16Q3	16Q4	17Q1	17Q2	17Q3	17Q4	18Q1
Berlin	9.622	9.479	9.590	9.818	9.044	9.507	9.694	9.484	9.545	9.114	8.926	8.494	8.733	8.781	8.885	9.222
	(0.011)	(0.010)	(0.007)	(0.007)	(0.010)	(0.007)	(0.007)	(0.008)	(0.007)	(0.008)	(0.008)	(0.008)	(0.007)	(0.007)	(0.006)	(0.009)
Cologne	9.785	9.683	9.773	9.983	9.199	9.654	9.892	9.773	9.817	9.303	9.092	8.680	8.939	8.988	9.102	9.458
	(0.020)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012	(0.011)	(0.011)	(0.012)	(0.013)	(0.014)	(0.013)	(0.013)	(0.012)	(0.012)	(0.016)
Düsseldorf	10.033	9.955	10.031	10.187	9.353	9.801	10.055	9.904	9.961	9.552	9.341	8.776	8.936	9.070	9.268	9.643
	(0.025)	(0.017)	(0.014)	(0.018)	(0.020)	(0.020)	(0.015)	(0.017)	(0.020)	(0.017)	(0.024)	(0.020)	(0.020)	(0.016)	(0.015)	(0.021)
Frankfurt	9.703	9.619	9.720	9.930	9.138	9.578	9.799	9.683	9.737	9.223	9.004	8.585	8.854	8.939	9.081	9.450
	(0.022)	(0.014)	(0.013)	(0.014)	(0.014)	(0.014)	(0.013)	(0.013)	(0.014)	(0.014)	(0.016)	(0.015)	(0.014)	(0.012)	(0.012)	(0.017)
Hamburg	8.384	8.337	8.426	8.606	7.850	8.317	8.507	8.352	8.409	7.943	7.741	7.280	7.495	7.544	7.647	7.965
	(0.019)	(0.012)	(0.011)	(0.013)	(0.013)	(0.013)	(0.011)	(0.012)	(0.013)	(0.012)	(0.016)	(0.013)	(0.013)	(0.011)	(0.010)	(0.014)
Munich	7.919	7.825	7.920	8.131	7.341	7.766	7.959	7.836	7.899	7.373	7.084	6.540	6.693	6.721	6.848	7.220
	(0.015)	(0.010)	(0.009)	(0.011)	(0.011)	(0.012)	(0.010)	(0.010)	(0.012)	(0.011)	(0.015)	(0.012)	(0.012)	(0.010)	(0.009)	(0.013)
Stuttgart	8.391	8.271	8.337	8.522	7.723	8.131	8.313	8.252	8.299	7.615	7.338	7.057	7.367	7.257	7.270	7.673
	(0.038)	(0.025)	(0.024)	(0.028)	(0.028)	(0.028)	(0.027)	(0.026)	(0.029)	(0.029)	(0.038)	(0.030)	(0.028)	(0.025)	(0.025)	(0.035)
Bochum	8.225	8.235	8.394	8.613	7.787	8.173	8.334	8.135	8.095	7.491	7.197	6.739	7.039	7.215	7.398	7.739
	0.068	0.041	0.036	0.042	0.046	0.043	0.035	0.038	0.042	0.041	0.052	0.05	0.048	0.041	0.038	0.055
Bonn	9.629	9.551	9.628	9.785	8.955	9.394	9.642	9.527	9.571	9.06	8.839	8.396	8.607	8.609	8.711	9.089
	0.036	0.023	0.02	0.023	0.024	0.023	0.02	0.021	0.022	0.021	0.024	0.024	0.024	0.022	0.021	0.03
Bremen	7.228	7.241	7.407	7.637	6.834	7.319	7.654	7.606	7.697	7.223	6.995	6.507	6.708	6.776	6.948	7.372
	0.058	0.035	0.031	0.034	0.038	0.037	0.031	0.033	0.036	0.033	0.04	0.038	0.037	0.03	0.027	0.037
Dortmund	8.515	8.467	8.503	8.572	7.708	8.156	8.377	8.127	8.156	7.856	7.653	6.917	6.944	7.138	7.396	7.76
	0.05	0.033	0.029	0.039	0.044	0.045	0.029	0.034	0.04	0.031	0.045	0.039	0.04	0.033	0.03	0.042
Dresden	6.454	6.269	6.283	6.441	5.651	6.128	6.368	6.184	6.168	5.66	5.449	4.995	5.213	5.263	5.429	5.87
	0.024	0.015	0.013	0.016	0.019	0.019	0.013	0.015	0.017	0.014	0.02	0.018	0.018	0.014	0.012	0.017
Duisburg	7.445	7.13	7.103	7.227	6.207	6.559	6.863	6.47	6.301	6.029	5.89	5.172	5.343	5.833	6.239	6.572
	0.075	0.048	0.041	0.053	0.06	0.062	0.04	0.045	0.052	0.045	0.06	0.049	0.05	0.042	0.039	0.058
Essen	8.191	7.978	7.96	8.065	7.187	7.613	7.888	7.777	7.78	7.2	6.934	6.518	6.854	7.048	7.24	7.59
	0.047	0.03	0.027	0.032	0.036	0.035	0.028	0.03	0.033	0.031	0.04	0.035	0.033	0.028	0.026	0.039
Hanover	7.599	7.565	7.7	7.929	7.156	7.621	7.875	7.79	7.871	7.38	7.196	6.821	7.111	7.171	7.261	7.565
	0.04	0.025	0.022	0.023	0.024	0.023	0.021	0.021	0.022	0.022	0.025	0.023	0.022	0.019	0.018	0.026
Karlsruhe	8.24	8.152	8.243	8.428	7.588	7.99	8.215	8.148	8.262	7.784	7.577	7.15	7.389	7.432	7.555	7.932
	0.046	0.029	0.026	0.028	0.03	0.029	0.027	0.027	0.029	0.029	0.034	0.033	0.031	0.027	0.025	0.035
Leipzig	5.851	5.77	5.829	5.952	5.098	5.653	6.075	5.87	5.857	5.594	5.548	5.047	5.175	5.228	5.423	5.882
	0.034	0.024	0.02	0.025	0.027	0.026	0.018	0.021	0.025	0.019	0.026	0.021	0.021	0.017	0.016	0.021
Mannheim	7.821	7.746	7.792	7.903	7.076	7.584	7.91	7.786	7.827	7.396	7.221	6.746	6.946	7.013	7.161	7.541
	0.051	0.032	0.028	0.033	0.035	0.034	0.027	0.029	0.032	0.028	0.035	0.031	0.03	0.025	0.024	0.033
Münster	9.841	9.942	10.05	10.14	9.343	9.845	10.08	9.887	9.887	9.422	9.233	8.763	8.955	8.998	9.19	9.679
	0.053	0.031	0.027	0.033	0.036	0.035	0.028	0.031	0.036	0.032	0.041	0.036	0.036	0.031	0.029	0.04
Nuremberg	6.577	6.566	6.687	6.893	6.144	6.532	6.603	6.507	6.583	5.914	5.624	5.315	5.632	5.58	5.637	6.044
	0.044	0.029	0.026	0.03	0.03	0.031	0.028	0.029	0.033	0.033	0.044	0.035	0.033	0.028	0.027	0.036
Wiesbaden	9.206	9.205	9.344	9.56	8.796	9.244	9.437	9.313	9.37	8.846	8.636	8.241	8.476	8.449	8.534	8.93
	0.043	0.027	0.024	0.027	0.029	0.029	0.024	0.025	0.027	0.026	0.034	0.03	0.028	0.024	0.023	0.031

Notes: (i) The table reports expected real capital gains and in brackets their standard deviations for 21 A- and B-cites in Germany. (ii) We assume that the housing market is in equilibrium and set $\omega_1 = 3.0\%$, $\delta = 2.5\%$, and $\gamma = 2.0\%$.

Our inflation series is based on the Consumer Price Index for Germany.[16] During the sample period, inflation had a minimum of -0.05% in 2014Q4 and a maximum of 1.64% in 2016Q4.

Summarizing all the assumptions, we get expected real capital gains as:

$$g_t^{real} = r_t + \omega_t + \delta_t + \gamma_t - \frac{R_t}{P_t} - \pi_t^e \tag{12}$$

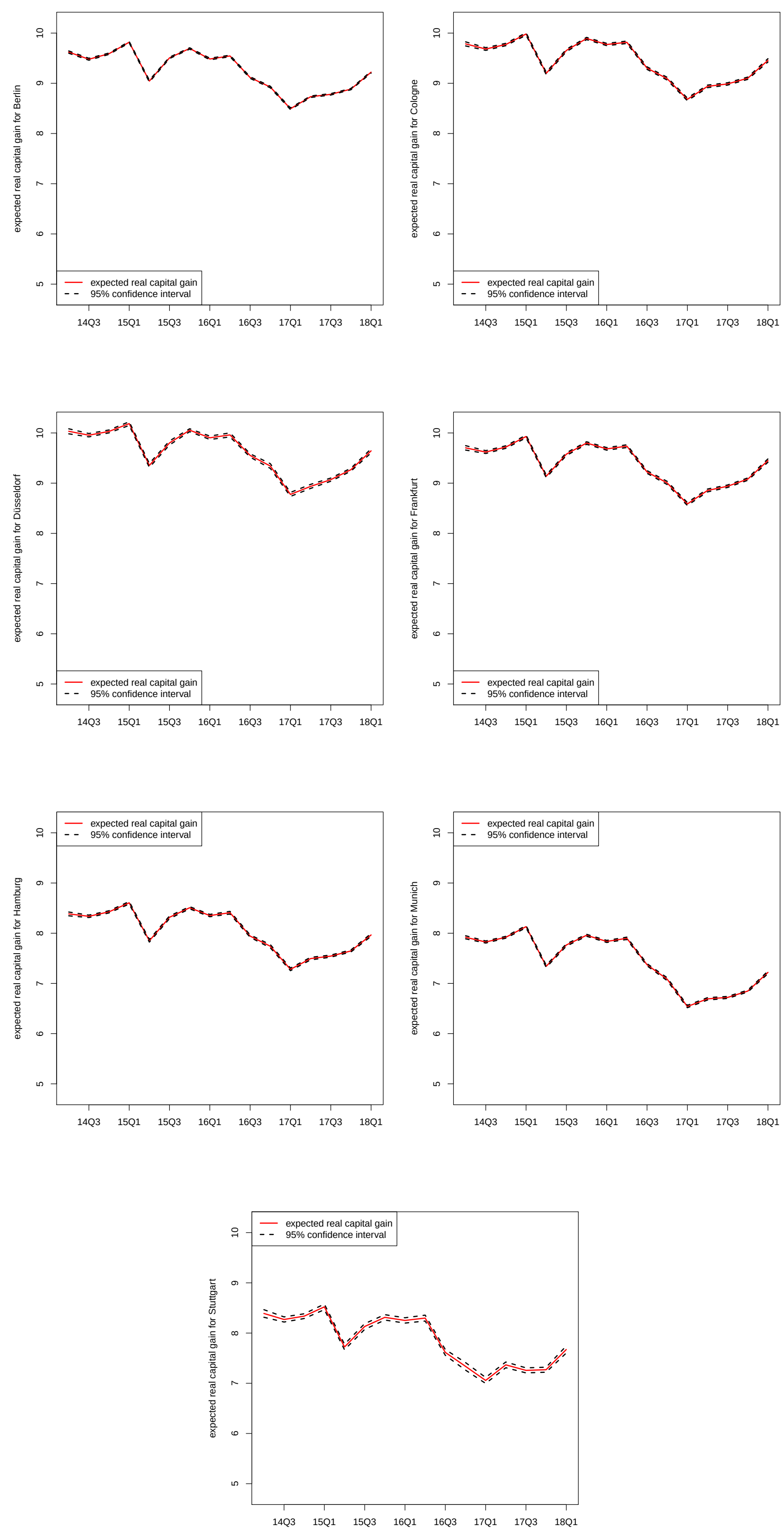
$$= r_t + \omega_1 + 3.0\% + 2.5\% + 2.0\% - \frac{R_t}{P_t} - \pi_{t-1}. \tag{13}$$

Results of our procedure are presented in Table 6. Figure 7 shows the development of expected real capital gains for the seven A-cities, and Figures 8-9 for the B-cities.

In absolute terms, we find the highest value for expected real capital gains (in 2018Q1) for Münster (9.7%), followed by Düsseldorf (9.6%), Cologne (9.5%), Frankfurt (9.5%), Berlin (9.2%), and Bonn (9.1%). Surprisingly, for Munich (7.2%) we find the lowest value of the A-cities. The lowest expected real capital gains are found for Dresden (5.9%).

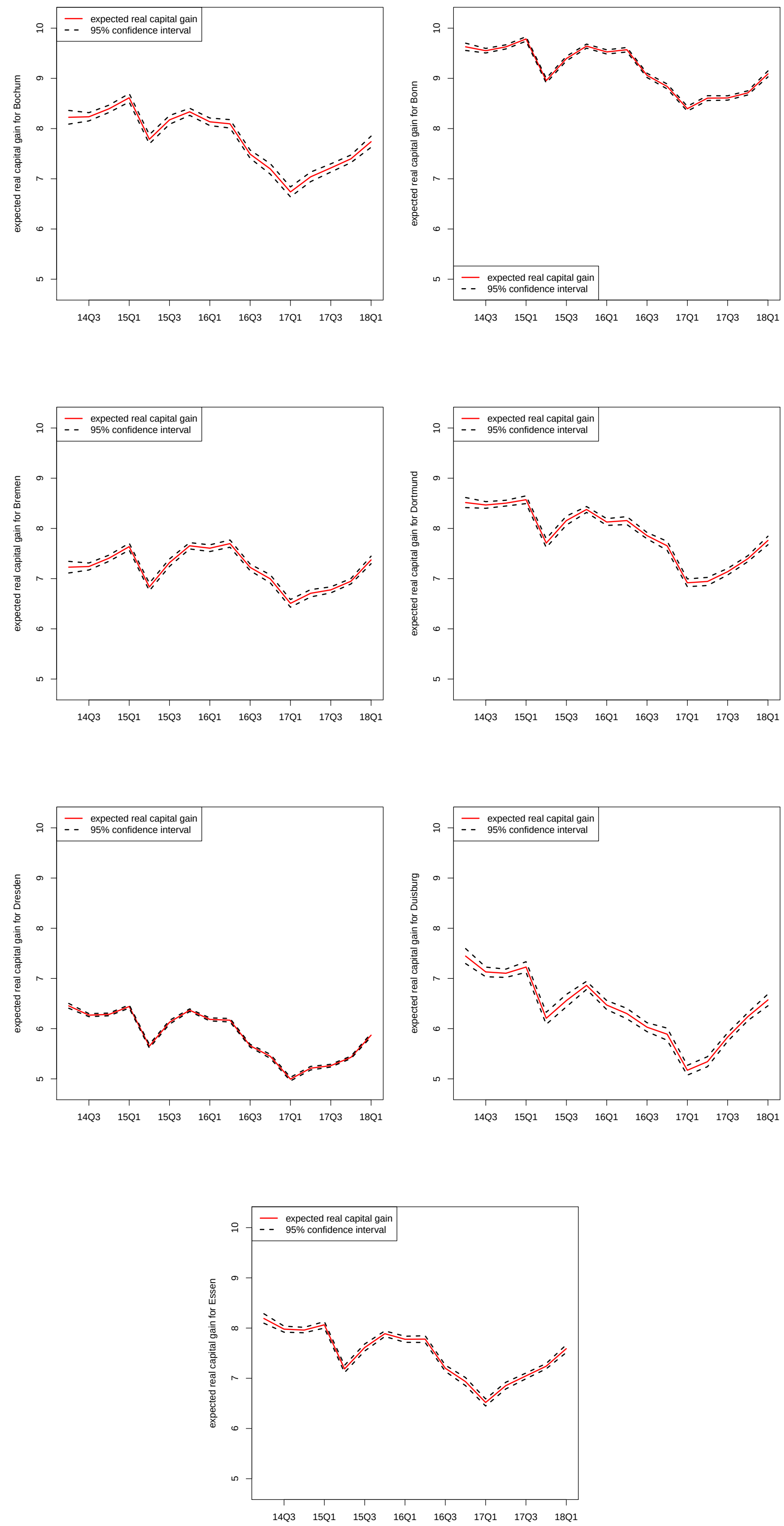
[16]Downloaded from `https://fred.stlouisfed.org/series/DEUCPIALLQINMEI`.

Figure 7: Expected real capital gain (in percent) for A-cities over the period 2014Q2 to 2018Q1.



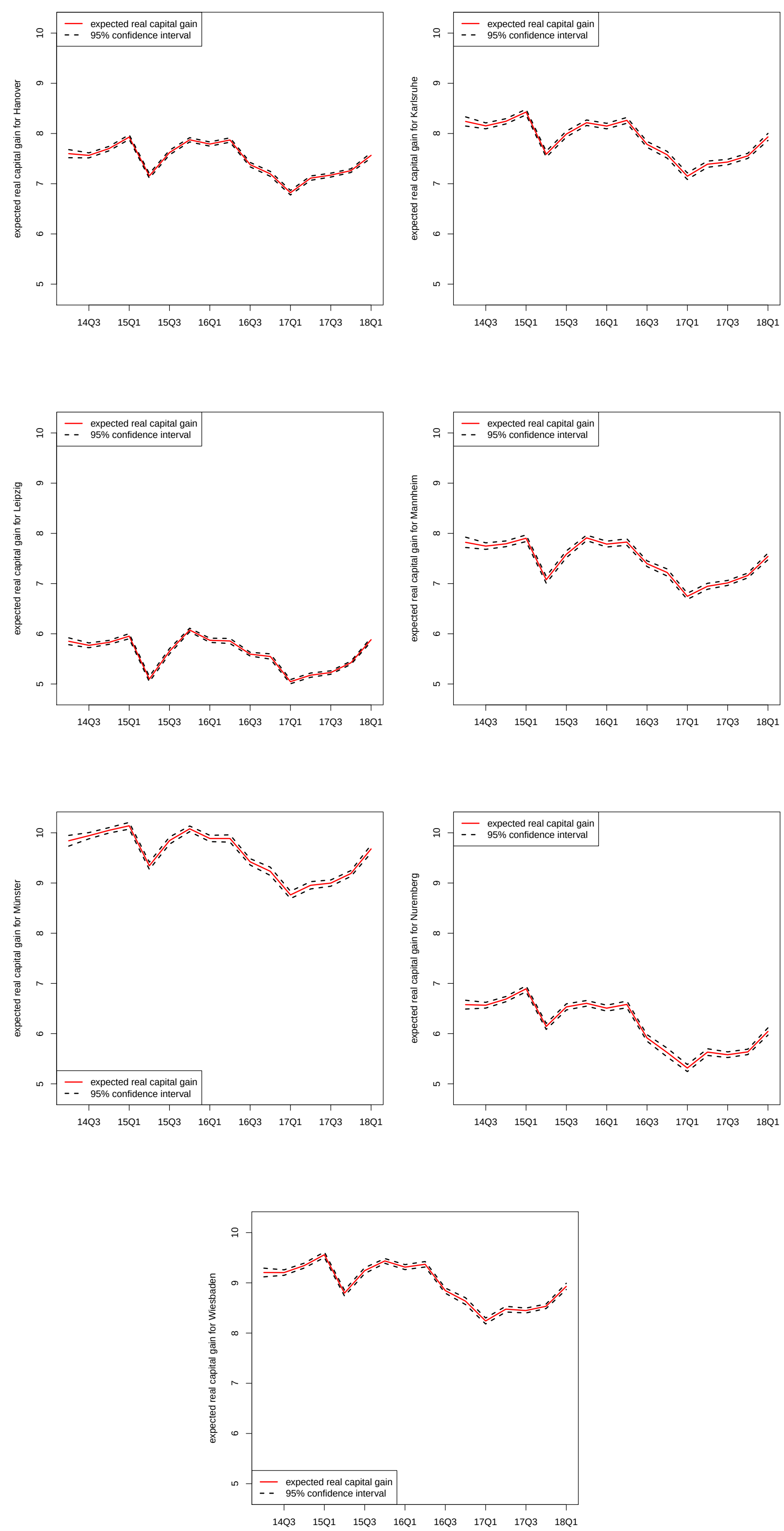
Notes: (i) The figure displays expected real capital gain (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for A-cities are shown: Berlin, Cologne, Düsseldorf, Frankfurt, Hamburg, Munich, and Stuttgart (linewise beginning at the top left panel)

Figure 8: Expected real capital gain (in percent) for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays expected real capital gain (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Bochum, Bonn, Bremen, Dortmund, Dresden, Duisburg, and Essen (linewise beginning at the top left panel)

Figure 9: Expected real capital gain (in percent) for B-cities over the period 2014Q2 to 2018Q1.



Notes: (i) The figure displays expected real capital gain (red solid line) together with its 95% confidence interval (dark dashed lines), (ii) Only results for B-cities are shown: Hanover, Karlsruhe, Leipzig, Mannheim, Münster, Nuremberg, and Wiesbaden (linewise beginning at the top left panel)

Over time, most cities experienced a decrease in expected real capital gain until 2017Q1 followed by a marginal recovery. The relative change over the full sample period is only for Bremen (2.0%) and Leipzig (0.5%) positive. For the other cities the expected real capital gain changed by -11.7% (Duisburg) to -0.4% (Hanover).

5 Conclusion

The importance of the residential real estate asset class has been widely demonstrated, for example, by a rampant sales price growth in recent years around the world, persevering governments that try to control rents and keep housing affordable for tenants, or the global financial crisis of 2007-11, which had its origin in the U.S. housing market. Unfortunately, our understanding of it is still limited. The purpose of this paper has been to show how quality-adjusted price-rent ratios can be constructed from micro data using a joint model for sales prices and rents. As a by-product of the modeling process, quality-adjusted property price and rental indices are obtained. With the quality-adjusted price-rent ratios it is then possible to estimate expected real capital gains. This enables us to answer important investment questions.

Using advertised asking rents and sales prices from 21 major German cities, we estimated quality-adjusted price-rent ratios and the expected real capital gain for apartments from 2014Q2 to 2018Q1. Our results show that there is a degree of heterogeneity across cities and time. We find price-rent ratios between 15.0 and 33.2, and expected real capital gain between 5.9% to 9.7% in 2018Q1. The increase in the sales price was between 13.5% and 60.5% over the sample period, while the increase in the rents was more moderate between 10.7% and 48.7%.

Overall, the approach introduced and applied to German data gives useful insights to prospective real-estate investors or market participants which have to decide whether to rent or buy an apartment.

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