The Productivity-Welfare Linkage: A Statistical Decomposition

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1.Introduction

In this paper I propose to examine the relationship between welfare and productivity. I plan to employ a statistical decomposition relating these two concepts. I will use measures of the two concepts which are available in national accounts. The data will be primarily for the United Kingdom but I hope to extend the analysis to other countries such as the United States.

According to Paul Krugman (1994, chapter 1), "Productivity isn't everything, but in the long run it is almost everything. A country's ability to improve its standard of living over time depends almost entirely on its ability to raise its output per worker." Though this seems intuitively obvious, the link between the two concepts is not straightforward. A standard measure of productivity is GDP per hour worked (an improvement on Krugman's GDP per worker). A reasonable measure of what might be termed economic welfare is median household income per equivalent adult (Office for National Statistics (2014), (2016) and (2017)). Median not mean, since we are interested in the experience of the typical household and the mean may be distorted by the gains accruing to the rich (the top 1% or top 0.1%). We may also be interested in the welfare of other groups, say the bottom 20% or 5%. And in a welfare context household income per equivalent adult is better than just household income since households differ in size and by whether or not they contain children; use of an equivalence scale allows for the different needs of different groups (Anyaegbu (2010)).

Of course, welfare in the broad sense is multi-dimensional and amongst the aspects excluded from purely economic welfare as defined here are health, longevity, leisure, personal freedom and autonomy, a fulfilling emotional life, and economic security. I do not pursue these issues here but focus solely on economic welfare.

2.The decomposition

Suppose on the welfare side that the income measure we are focusing on is Median Equivalised Household Disposable Income ($EHDI^{median}$) as defined above. Denote the corresponding arithmetic mean by $EHDI^{mean}$. Let us take our aggregate productivity measure to be GDP per capita (below we extend the decomposition to GDP per *hour*).

The transformation between productivity on the right hand side and welfare on the left hand side can be written as the product of a set of factors as follows:

$$EHDI^{median} \equiv \left(\frac{EHDI^{median}}{EHDI^{mean}}\right)\left(\frac{EHDI^{mean}}{HDI\,/\,N}\right)\left(\frac{HDI}{GDP}\right)\left(\frac{GDP}{N}\right) \qquad (1)$$

This relates median EHDI through a series of factors to GDP per capita (*GDP*/*N*) where *N* is the population. This is just an identity but the factors can be given an economic interpretation and also can be tracked over time. The factors are

$\frac{EHDI^{median}}{EHDI^{mean}}$: a measure of inequality. If this rises, inequality is falling. If the distribution of *EHDI* is (approximately) lognormal, then this ratio equals $\exp(-\sigma^2/2)$ where σ^2 is the variance of log income.

$\frac{EHDI^{mean}}{HDI/N}$: a measure of household composition. The numerator is the arithmetic mean of equivalised HDI while the denominator is the mean of HDI without equivalisation, where *N* is population. If household size is rising due to more single persons partnering up, with other things the same, then the mean of the equivalised HDI rises in relation to the mean of the unequivalised distribution.

$\frac{HDI}{GDP}$: a measure of the size of the welfare state. The ratio rises if the government spends more on transfers or reduces tax. It also rises if net foreign income accruing to households rises as a proportion of GDP.

So far the decomposition is much the same as the one employed by Nolan et al. (2018). But we are not quite ready yet to analyse Krugman's contention since the right hand side of equation (1) features GDP per capita not GDP per hour. These two concepts can be linked through a second identity:

$$\frac{GDP}{N} \equiv \left(\frac{H}{N}\right)\left(\frac{GDP}{H}\right) \equiv \left(\frac{H}{(1-u)L}\right)(1-u)\left(\frac{L}{N^{wa}}\right)\left(\frac{N^{wa}}{N}\right)\left(\frac{GDP}{H}\right) \qquad (2)$$

where *H* is aggregate hours worked, *L* is the number of people in the labour force (employed plus unemployed), *u* is the unemployment rate, and N^{wa} is the number of people of working age (defined say as those aged between 18 and 64). The factors on the right hand side can be given the following interpretation:

$\frac{H}{(1-u)L}$: hours per worker, or labour intensity.

$1-u$: 1 minus the unemployment rate (*u*).

$\frac{L}{N^{wa}}$: the labour force participation rate.

$\frac{N^{wa}}{N}$: the proportion of the population which is of working age, or 1 minus the dependency ratio.

This decomposition is for median household income but could be adapted for any other quantile, such as equivalised HDI at the lowest quintile, the poorest fifth, or $EHDI^{quin1}$. With $EHDI^{quin1}$ on the left hand side the first ratio on the right hand side can then be changed to

$$\frac{EHDI^{quin1}}{EHDI^{mean}}$$

which can also be interpreted as a measure of inequality.

This decomposition applies at a point in time or in other words income and output are in current prices. But the main interest is in tracking changes over time, i.e. we want to relate real HDI to real GDP. Real GDP is related to nominal GDP by the implicit GDP deflator, P^{GDP}, while real HDI is related to nominal HDI by an index of consumer prices, P^{CE}. The full decomposition now becomes

$$\frac{EHDI^{median}}{P^{CE}} \equiv \left(\frac{P^{GDP}}{P^{CE}}\right)\left(\frac{EHDI^{median}}{EHDI^{mean}}\right)\left(\frac{EHDI^{mean}}{HDI/N}\right)\left(\frac{HDI}{GDP}\right)\left(\frac{H}{(1-u)L}\right)(1-u)\left(\frac{L}{N^{wa}}\right)\left(\frac{N^{wa}}{N}\right)\left(\frac{GDP}{P^{GDP}H}\right) \quad (3)$$

Now we have introduced an eighth factor:

$\frac{P^{GDP}}{P^{CE}}$: the price of GDP as a whole relative to the price of consumption. This can be thought of as reflecting technological trends, i.e. relative rates of productivity growth in different industries. For example the price of investment goods may be falling relative to that of consumption goods but the opposite may be the case for government services such as health and education (at least as conventionally measured).

The (logarithmic) growth rate of living standards can now be thought of as the sum of the growth rates of the nine factors on the right hand side of equation (3). So equation (3) yields an additive decomposition relating the growth of living standards to the growth of productivity.

There are a number of ways in which the decomposition could be expanded. For example, the measure of the size of the welfare state, HDI/GDP, could be broken down further to show the separate contributions of taxes and benefits to changes in this ratio. And productivity growth itself could be broken down into the contributions of TFP and capital deepening.

Conclusion

A statistical decomposition has been developed to relate the growth of economic welfare, measured by median household disposable income per equivalised adult, to the growth of productivity, measured by GDP per hour worked. The decomposition involves eight factors, each of which can be given an economic or demographic interpretation.

The plan is to implement the decomposition using data for the UK. If time allows other countries such as the US could be added.

A decomposition by itself cannot explain anything. But it can be used as a diagnostic tool. If all the factors except productivity stayed constant then welfare and productivity would grow at the same rate. Or we might find that the factors are all changing, but in an offsetting fashion. Or the growth of the factors taken together may impart an upward or downward movement to welfare relative to productivity. If the latter, then further investigation of the causes of changes in the factors would be warranted.

References

Anyaegbu, Grace (2010). "Using the OECD equivalence scale in taxes and benefits analysis". *Economic & Labour Market Review*, vol. 4 (1), January. webarchive.nationalarchives.gov.uk/20160105160709/http:/www.ons.gov.uk/ons/rel/elmr/economiconomic-and-labour-market-review/no--1--january-2010/using-the-oecd-equivalence-scale-in-taxes-and-benefits-analysis.pdf.

Krugman, Paul (1994). *The Age of Diminished Expectations*. Cambridge: The MIT Press.

Nolan, Brian, Max Roser, and Stefan Thewissen (2018). "GDP per capita versus median household income: what gives rise to the divergence over time and how does this vary across OECD countries?". *Review of Income and Wealth*, Series 65, Number 3, September, pages 465-494.

Office for National Statistics (2016). "A guide to sources of data on earnings and income update". https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/earningsandworkinghours/methodologies/aguidetosourcesofdataonearningsandincome.

Office for National Statistics (2014). "Economic well-being: framework and indicators" http://webarchive.nationalarchives.gov.uk/20160105160709/http://www.ons.gov.uk/ons/rel/wellbeing/economic-well-being/framework-and-indicators/art-economic-wellbeing.html.

Office for National Statistics (2017). "Household disposable income and inequality in the UK: financial year ending 2016". https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/bulletins/householddisposableincomeandinequality/financialyearending2016.