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Net Operating Surplus for Government-Owned Assets:

The Case for a User-Cost Basis, and Preliminary Annual Estimates

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0. Abstract

This essay argues for a full-cost accounting of fixed assets owned by governments. Present SNA practice countenances capital consumption only, but not net operating surplus. Put another way, the net own rate of return to government-owned assets is set to zero, independent of macroeconomic conditions or the performance of public managers. That approach is mistaken on normative and positive grounds. Discussions of the treatment of publicly-held assets have long been informed by two conflicting views: on the one hand, government activities cannot pass a market-test and so merit a zero net return at most; on the other, returns to assets used in government activities likely mimic such assets' private-sector returns. Both views make the same normative error that government activities are comparable to private ones. Giving up the hope of comparability allows this paper to take a middle view: government activities indeed are often not similar to private ones, but they are (loosely) disciplined by bond markets; moreover, government program managers face internal incentives against gross waste. The two disciplinary devices suggest that estimating implicit government returns based on a user-cost concept is more reasonable than imposing a zero net rate. Further, government debt-auction results and investment-goods deflators are readily available, as are implicit depreciation rates, so constructing user-costs for publicly owned assets is straightforward.

The positive case comes down to implementation details. First, in view of the public debt's yield curve, the paper presents a toy model showing that immediate *forward*-rates are appropriate for the user-cost, while forward-averaging *yield*-rates apply to discounting future service-flows. (The argument is the same as using the earliest depreciation rate, not an average, in the user-cost when consumption of fixed capital is nongeometric.) Second, Kalman filter/smoother treatment of investment-goods price-inflation successfully distinguishes smooth paths of expected revaluations from noise (the relative proportions differ among asset types); the resulting update formulas allow genuine forecasts, so that the commonplace use of last year's price inflation in this year's user-cost, is no longer necessary. None of this is out of reach for compilers of national accounts. A demonstration of the approach on U.S. data since 1987 shows that a nonzero central government net operating surplus makes a noticeable difference on measures of overall output, increasing nominal GDI as much as $89 billion in 1989, while reducing it by $76 billion in 2010. (This version of the paper does not consider the net operating surplus of state and local governments, which finance themselves in the U.S. federal system and so may or may not face the same yield curve as the central government.) In fact, incorporating a nonzero own rate of return would have reduced nominal GDI in all but two years since 2002, as the U.S. economy entered a liquidity-trap.

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1. Introduction: The Normative Case

The 2008 System of National Accounts offers scant support for the treatment of government-owned capital symmetrically with nonfinancial assets owned and used by the private business sector.[1] According to the SNA, nonpriced outputs (the usual case for government) are to be valued at production costs, and the production costs assigned to capital include depreciation only, as the net rate of return to government capital is set to zero. This rather circular restriction places puzzling demands on the accounting treatment of public-sector productive capital, for non-zero interest rates are regularly updated for use in evaluating public-sector purchases (OMB, 2019).

One may trace several reasons for objecting to net government-asset returns. The first is that net returns are somehow sullied, as they may include economic profits or losses (which ought not be the government's concern) in addition to the straightforward cost of capital. Yet excluding the entire net return also denies the cost of capital (which might have disciplined governmental decisions). Perhaps avoiding the market's discipline is the point. The second is that states are inherently wasteful, any assets the government might own would fail a market test, so a zero return is the best hope. A third, conspiratorial reason is that governments do indeed optimize, on behalf of the shadowy forces that pull the levers, so it is best not to let the hoi polloi see how well the government might instead do for *them*. All three views (and perhaps others) disagree with each other, but all agree on a zero rate. I will not speculate on their relative strengths in informing SNA section 6.130.

Not all are happy with a zero net return to public capital. The Office of Productivity and Technology of the U.S. Bureau of Labor Statistics assigns a rate of return to government-owned assets that is derived from the implied returns to assets held by the private sector. This faces the immediate objection that governments do not, and in many ways should not, operate like businesses, and certainly do not respond to business cycles as private companies do. Academic studies sometimes find external productive effects attributable to government activities — see Aschauer (1989), Munnell (1990a, 1990b), and many others since — though the think-tank response should not surprise (Mitchell, 2005). Mass-producing such studies for comprehensive and timely official statistics would be a very large undertaking, liable to bog down in econometrics.

A more transparent approach would carefully apply the user-cost concept to government-owned capital. Unlike the first of the three zero-rate rationales listed above, it presumes an optimization context. Unlike the second (and the BLS method), it does not link government returns to those of business. And unlike the third, it is agnostic about the goals of optimization. The user-cost approach strips out economic profits or losses to report unsullied net returns, be they positive or negative (in addition to the existing capital consumption component). This paper will report cases of both.

[1] See §6.130 especially (p. 111). Only §A4.17 (p. 604) shines some light toward alternative treatments, which is a research item this paper attempts to enlighten.

2. Net Operating Surplus and Discounting: Toy Stories

From the viewpoint of the user-cost, the net operating surplus is intimately connected to discounting. Under the competitive optimization framework commonly assumed of private businesses (and which this proposal would extend to government operations), the difference between the nominal interest rate on committed funds and the expected revaluation rate of the price of the relevant new investment-good type — commonly termed the own rate-of-return — is used to discount future earnings attributable to the asset-type, and the product of the own rate-of-return and the type's nominal net stock, summed across all types, should approximate the operating unit's Net Operating Surplus (NOS).[2] For governments, whose services are often available for free or a reduced price, NOS cannot be estimated as an actual accounting surplus, but would need to be calculated as described. The well-known dual relation in production economics between an asset's resale and rental values only works if the discount rate and NOS correspond; to the extent that governments discount the future at all realistically in their capital spending decisions, they cannot not have a non-zero net operating surplus.

To see this, consider the following toy model of an s-year-old individual asset j whose resale value declines parabolically as it ages, vis-à-vis the price of an otherwise-identical new asset:

$$P_s/P_0 = \left(1 - \frac{t-b}{L_j}\right)^2 \qquad \text{... for } b \le t \le b+L_j\text{, else } 0 \qquad (1)$$

...where s is the individual's age (that is, the difference between the current date t and the date b when the individual began service) and L_j is the individual's service-life.[3] The parabolic form is the continuous limit of the sum-of-years'-digits depreciation pattern from elementary accounting.[4]

A standard rental-price derivation gives the age-s rental price R_s:

$$R_s/P_0 = \left(1 - \frac{t-b}{L_j}\right)^2 \left(r + \frac{2}{L_j-(t-b)}\right) \quad \text{... for } b \le t \le b+L_j\text{, else } 0 \qquad (2)$$

...where the own rate-of-return $r = i - \partial ln\mathrm{P}_0/\partial t$. The individual user-cost follows (set $s = 0$, or $t = b$) as:

$$U_j = P_0\left(r + \frac{2}{L_j}\right) \qquad (3)$$

...so the individual depreciation rate starts out as double declining-balance.

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[2] If NOS falls short of the calculation for too long, economists expect a private business to contract, but the same would not necessarily hold for a government, where "Baumol's Rule" predicts the migration of activities no longer privately profitable. If NOS exceeds the calculation for too long, the state's antitrust authorities grow suspicious, private businesses plead (not-so-) temporary gains from innovations, and economists search for asset-types they might have missed.

[3] It is unavoidable that other individuals from the same batch/cohort/model-year as *j* will have different individual service-lives. The distribution of service-lives across individuals is of independent interest. Relative frequencies of individual lives serve as weights in the computation of cohort-level resale-price profiles, rental-price profiles, and user-costs. The age-efficiency profile follows as the ratio of the rental-price profile to the user-cost.

[4] Where accounting and production economics differ is in their conceptions of the *new* price, P_0. In the former, it is the price of the now *s*-year-old asset *s* years ago, when it was new, so that depreciation kills off the original investment after L_j years. In the latter, which seeks to convert old capital to current new equivalents, it is the price of a new asset now, at date *t*. Converting between the two conceptions of P_0 requires knowing the asset-type's constant-quality inflation rate and the rate of quality change across cohorts.

The net present value integral restores the resale-price profile from the rental-price profile:

$$P_s/P_0 = \int_t^{b+L_j} e^{-r(u-t)} \left(1-\frac{u-b}{L_j}\right)^2 \left(r+\frac{2}{L_j-(u-b)}\right) du \qquad (4)$$
$$= \left(1-\frac{t-b}{L_j}\right)^2$$

...where the variable of integration u runs over dates from t (i.e., now) through $b+L_j$ (j's retirement). Setting $t = b$ implies $P_s/P_0 = 1$.

The integral (4) also works if $r = 0$ in both the discounter and the rental price, but the market for government bonds certainly behaves as if $r \neq 0$, so we should take discounting seriously. At the same time, current SNA guidance would impose $r = 0$ in the net operating surplus. What then?

$$P_0^{Govt}/P_0 = \int_b^{b+L_j} e^{-r(u-b)} \left(1-\frac{u-b}{L_j}\right)^2 \left(0+\frac{2}{L_j-(u-b)}\right) du \qquad (5)$$
$$= \frac{2}{r\,L_j}\left(1-\frac{1-e^{-r\,L_j}}{r\,L_j}\right) \leq 1$$

To make a new government-owned asset worth its purchase price, i.e., $P_0^{Govt}/P_0 = 1$, we must either set $r = 0$ in the discounter, implying governmental farsightedness that conflicts with both the bond market and the newspaper, or drastically shorten the service life $L_j \to 0$, which seems more consistent with no-tomorrow governance. Nothing in between.

Toy models are dismissible as unrealistic. Practical national accountants have neither the time nor (in most countries) the data to consider individual assets. So statistical services aggregate across individuals to the cohort / model-year level, which is more in keeping with the data available to the agencies. Fortunately, a Gamma density of service-lives with shape parameter 3 and rate parameter δ:

$$f_L(L) = \frac{1}{2}\boldsymbol{\delta^3 L^2 e^{-\delta L}} \qquad (6)$$

...which has a well-defined interior mode and a long right tail (in reasonable agreement with actual studies of service-life distributions), provides just the weights to aggregate individual-level expressions (1)-(5) to recognizable geometric cohort-level counterparts:[5]

$$\bar{P}_s/P_0 = \int_{t-b}^{\infty} \left(1-\frac{t-b}{L}\right)^2 \times \frac{1}{2}\boldsymbol{\delta^3 L^2 e^{-\delta L}}\, dL = e^{-\delta(t-b)} \qquad (7)$$

$$\bar{R}_s/P_0 = \int_{t-b}^{\infty} \left(1-\frac{t-b}{L}\right)^2 \left(r+\frac{2}{L-(t-b)}\right) \times \frac{1}{2}\boldsymbol{\delta^3 L^2 e^{-\delta L}}\, dL = (r+\delta)e^{-\delta(t-b)} \qquad (8)$$

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[5] For an individual-level resale-price profile of form $P_s/P_0 = \left(1-\frac{s}{L}\right)^{v-1}$, a Gamma density of service-lives $f_L(L) = \delta^v L^{v-1} e^{-\delta L}/\Gamma(v)$, with $v > 0$ and $\delta > 0$, provides the exact weights to satisfy $e^{-\delta s} = \int_s^{\infty} P_s/P_0\,(s,L) \times f_L(L)\,dL$. It is left to the reader to decide whether $v \leq 2$ is reasonable. *Cf.* Sliker (2018).

$$\bar{U} = P_0 \int_0^\infty \left(r + \frac{2}{L}\right) \times \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\delta}^3 \boldsymbol{L}^2 \boldsymbol{e}^{-\boldsymbol{\delta L}}\, dL = P_0 (r + \delta) \tag{9}$$

$$\bar{P}_s \big/ P_0 = \int_t^\infty e^{-r(u-t)} \times (r + \delta) e^{-\delta(u-b)} du = e^{-\delta(t-b)} \tag{10}$$

Of particular interest is the cohort-aggregate version of (5):

$$\overline{P_0^{Govt}} \big/ P_0 = \int_0^\infty \frac{2}{r\,L}\left(1 - \frac{1-e^{-r\,L}}{r\,L}\right) \times \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\delta}^3 \boldsymbol{L}^2 \boldsymbol{e}^{-\boldsymbol{\delta L}}\, dL = \frac{\delta}{r+\delta} \tag{11}$$

...which addresses the last-resort argument against assigning government-owned assets a non-zero net operating surplus: that doing so would be inappropriate because governments purchase assets at concessionary prices, in violation of the Law of One Price. In a setting of geometric cohort-depreciation, that just-so story faces a simple test: in markets where both private companies and the government are customers, do companies pay 100 r/δ percent more for the same goods as governments?

Given the principled reasonability of using a non-zero own rate-of-return to estimate government net operating surplus, what is the appropriate rate to use in practice? Here a second toy model — of a yield curve, in rough approximation of the government bond market — combined with the first, will provide an answer. Replace the constant-rate discounting function, $e^{-r(u-t)}$, by a simple weighted average:

$$D(u-t) = e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right) \quad \ldots h > l > 0,\ \alpha < 1 \tag{12}$$

Like $e^{-r(u-t)}$, the compound form tells the current (i.e., date-t) price of a unit of currency to be received u periods in the future. Unlike $e^{-r(u-t)}$, the compound form is not time-consistent, as it takes *some* account of anticipated changes in economic conditions, and there is no obvious "rate" like r. Minute-by-minute "forward" rates are found as:

$$fr(u-t) = -\frac{\partial\, ln\, D(u-t)}{\partial(u-t)} = \frac{l\, e^{-l(u-t)} + \alpha\left(h\, e^{-h(u-t)} - l\, e^{-l(u-t)}\right)}{e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)} \tag{13}$$

...and through-time average "yield" rates as:

$$yr(u-t) = -\frac{\ln D(u-t)}{u-t} = -\frac{ln\left[e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)\right]}{u-t}. \tag{14}$$

The two rates begin together (i.e., when $u = t$) at $l + \alpha(h - l)$, monotonically increase or decrease, and finish together (i.e., as $u \to \infty$) at l, but the yield as an average is less nimble than the forward. Rates increase over farther horizons when $\alpha<0$; they decrease for $0 < \alpha < 1$.[6]

[6] To set parameter values that deliver a desired set of curves, begin by assigning numerical values to the zero-horizon rate (i.e., $l + \alpha(h - l) = fr_0$) and the asymptotic rate (i.e., $l = fr_\infty$); also choose a middle rate fr_m for a medium horizon (say 3 years). For increasing curves, initialize $\alpha = -.5$; for decreasing curves, initialize $\alpha = .5$. Then set $h = l + \frac{fr_0 - l}{\alpha}$, then $\alpha = \frac{fr_m - l}{fr_m - l + (h - fr_m) Exp[-(h-l)3]}$, back-and-forth, repeatedly. Convergence takes about 10 rounds.

When the own rate-of-return is not constant over future horizons, derivations of rental-price profiles from resale (and the reverse) must be adjusted, as well as the user-cost. At the individual-asset level, transforming the resale-price profile (1) to its rental-price counterpart at any future time ($u \geq t$) involves replacing r by the u-periods-ahead-of-t *forward* rate:

$$R_{s+u-t}/P_0 = \left(1 - \frac{u-b}{L_j}\right)^2 \left(\frac{l\,e^{-l(u-t)} + \alpha\left(h\,e^{-h(u-t)} - l\,e^{-l(u-t)}\right)}{e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)} + \frac{2}{L_j-(u-b)}\right), \qquad (15)$$

...so the current-period individual user-cost (set s=0 and u=b=t) is:

$$U_j = P_0\left(l + \alpha(h-l) + \frac{2}{L_j}\right) \qquad (16)$$

On the other hand, the discounting function in the net present value integral uses the *yield* rate where it formerly used the constant r: Exp[$-yr \times (u-t)$]. The appropriate integral evaluates correctly:

(17)

$$P_s/P_0 = \int_t^{b+L_j}\left(e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)\right) \times \left(1 - \frac{u-b}{L_j}\right)^2 \left(\frac{l\,e^{-l(u-t)} + \alpha\left(h\,e^{-h(u-t)} - l\,e^{-l(u-t)}\right)}{e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)} + \frac{2}{L_j-(u-b)}\right) du$$
$$= \left(1 - \frac{t-b}{L_j}\right)^2 .$$

The Gamma-density weighting exercise averages across individuals to the cohort results, as before:

$$\bar{R}_{s+u-t}/P_0 = \int_{u-b}^{\infty}\left(1 - \frac{t-b}{L}\right)^2 \left(\frac{l\,e^{-l(u-t)} + \alpha\left(h\,e^{-h(u-t)} - l\,e^{-l(u-t)}\right)}{e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)} + \frac{2}{L-(t-b)}\right) \times \frac{1}{2}\boldsymbol{\delta}^3\boldsymbol{L}^2\boldsymbol{e}^{-\boldsymbol{\delta L}}\, dL$$
$$= \left(\frac{l\,e^{-l(u-t)} + \alpha\left(h\,e^{-h(u-t)} - l\,e^{-l(u-t)}\right)}{e^{-l(u-t)} + \alpha\left(e^{-h(u-t)} - e^{-l(u-t)}\right)} + \delta\right) e^{-\delta(u-b)} \qquad (18)$$

$$\bar{U} = P_0 \int_0^{\infty}\left(l + \alpha(h-l) + \frac{2}{L}\right) \times \frac{1}{2}\boldsymbol{\delta}^3\boldsymbol{L}^2\boldsymbol{e}^{-\boldsymbol{\delta L}}\, dL = P_0(l + \alpha(h-l) + \delta) \qquad (19)$$

(Expressions (7) and (10) are unaffected.) Whether at the individual or cohort level (or even the net stock of an asset type, which sums across cohorts), the models come to the same conclusion: the own rate-of-return to be used in calculating the net operating surplus for government-owned assets should be of the shortest feasible horizon.

3. Estimating Zero-Horizon Rates from the U.S. Treasury Yield Curves

This section describes the estimation of zero-horizon nominal interest rates based on the results of U.S. Treasury auctions, using readily available yield-curve data reported from the auctions themselves on a nearly daily basis since January 1990, as well as two academic studies that dig deeply into Treasury auctions as far back as June 1961. The subsequent section describes the application of the Kalman filter/smoother since 1987 to annual implicit investment-deflator inflation-rates of asset types owned by the U.S. federal government. Resulting (calendar) year-ahead inflation-rate forecasts are then subtracted from the year-averaged zero-horizon nominal interest rate estimates to form this paper's estimates of the own rate-of-return for each federally-owned asset type.

Near-daily U.S. Treasury bond *par-type* interest rate results are available since January 2, 1990, at https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield. Par rates include a semi-annual coupon payment priced at the same rate as the bond's interest rate. Par-type interest rates are convertible to the zero-coupon format of this paper's forwards and yields, but this is not necessary for the zero horizon, as all three formats coincide there. Treasury's published horizons, ranging from 1 month through 30 years, are available for different eras:

	1m	2m	3m	6m	1y	2y	3y	5y	7y	10y	20y	30y
1/2/90-9/30/93			✓	✓	✓	✓	✓	✓	✓	✓	✓	
10/1/93-7/30/01			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
7/31/01-2/15/02	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2/19/02-2/8/06	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	
2/9/06-10/15/18	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
10/16/18-present	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Two academic papers synthesize and extend the Treasury yield curve back into 1961. Gürkaynak, Sack, and Wright (2007), fit near-daily de-couponed bond-auction results to the well-known 6-parameter flexible form of Nelson and Siegel (1987) as extended by Svennson (1994). Their estimates are available at: https://www.federalreserve.gov/data/nominal-yield-curve.htm. While these are not official Federal Reserve products, their method has been adopted by several central banks. However, a recent kernel-averaging approach by Liu and Wu (2021) hews closer to the bonds (and bills) data. Monthly and daily results are available at: https://sites.google.com/view/jingcynthiawu/yield-data, and the authors have promised updates. I elected to use Liu and Wu's results for 1987-89, and the reported U.S. Treasury rates since 1990. I applied natural cubic splines to both sets of interest-rate profiles. Splines are highly regarded as interpolators, but their use as extrapolators is less common. However, my extrapolation back to the zero horizon was not far, and the results (in comparison to the overnight Federal Funds rate) are reasonable.[7] Year-averaged zero-horizon yields are below, in Table 1.

[7] In the case of equal spacing from the zero horizon onward, the natural cubic spline extrapolation of the nominal interest rate i back to the zero-horizon amounts to: $i_0 = 2i_{\text{earliest}} - i_{\text{second-earliest}}$.

Table 1: Very Short-Horizon Interest Rates

	Zero-Horizon Treasury Rate Estimates (%)[8]	Federal Funds Rate (%)
1987	5.34	6.66
1988	6.66	7.57
1989	8.17	9.21
1990	7.64	8.10
1991	5.39	5.69
1992	3.37	3.52
1993	2.91	3.02
1994	3.91	4.21
1995	5.50	5.83
1996	5.00	5.30
1997	5.01	5.46
1998	4.80	5.35
1999	4.61	4.97
2000	5.82	6.24
2001	3.53	3.88
2002	1.63	1.67
2003	1.02	1.13
2004	1.20	1.35
2005	2.89	3.22
2006	4.70	4.97
2007	4.37	5.02
2008	1.25	1.92
2009	0.07	0.16
2010	0.10	0.18
2011	0.04	0.10
2012	0.06	0.14
2013	0.04	0.11
2014	0.03	0.09
2015	0.03	0.13
2016	0.22	0.39
2017	0.80	1.00
2018	1.78	1.83
2019	2.12	2.16
2020	0.35	0.38

[8] 1987-89: extrapolated from the 1- and 2-month horizon estimates of Liu and Wu (2021), then averaged up to a yearly basis. 1990-2020: extrapolated from the earliest two evenly-spaced U.S. Treasury rates (3- and 6-months for 1/2/90-7/30/01, 1- and 2-months thereafter), then averaged up to a yearly basis.

4. Estimating Asset-Type Revaluations

This section describes the use of the Kalman filter to forecast the investment-price inflation-rates of asset types owned by the U.S. federal government. Subtracting the forecasts from the zero-horizon nominal interest rates estimated in the previous section gives each asset type's own rate of return. The twenty-five "types," drawn from six different groupings in the U.S. Fixed Assets Accounts, are already rather highly aggregated:

U.S. Federal Government Asset-Types

Defense Equipment	Non-Defense Equipment
Aircraft	Non-Defense Structures
Missiles	Office
Ships	Commercial
Vehicles	Health care
Electronics	Educational
Other Equipment	Public Safety
Defense Structures	Amusement and Recreation
Residential	Transportation
Industrial	Power
Military Facilities	Highways and Streets
Defense Intellectual Property Products	Conservation and Development
Software	Other Structures
Research and Development	Non-Defense Intellectual Property Products
	Software
	Research and Development

...nonetheless this paper treats each type as if it were a homogeneous good.[9] Deflators are implicit, formed as ratios of nominal investment by type (from the U.S. Fixed Assets Table 7.5) to quantity indexes by type (Table 7.6), then rebased to 1 in 2012. Year-to-year log-differences transform the deflators into symmetric inflation rates, which are the raw material for Kalman filtering. Inflation-rate series begin as early as 1917-18 (for Military Facilities) and as late as 1990-91 (for Non-Defense Power Structures), although I viewed the time-series as "raw material" for good estimates, being willing to drop visible outliers or early eras if rough stationarity seemed jeopardized for forecasts since 1987.

I adhered to Jones' (1980) Kalman filter formulation closely. This is an ARMA setup (i.e., AR1MA in *ln*-deflators) that also distinguishes observational errors from moving-average modeling errors. To discipline the filter and identify the observational error variance, I constrained the empirical variance of

[9] A further 17 types owned by states and localities has a combined net stock value exceeding that of the 25 types shown here (chiefly on the strength of schools and roads).

the innovations to equal the average of the parametrized innovation-variance terms (after a 1-period burn-in, which was somewhat short). I also constrained the point values of the relevant autoregressive and moving-average coefficients to stay within the interior 99 percent of their stationary and invertible spaces. To choose the proper model, I used Akaike's Information Criterion as corrected for small samples, instead of simply selecting the model with the greatest (assumed-Normal) constrained likelihood, which overfits. For each asset-type, I tested ten different models, ranging from zero-inflation expectations *versus* a verbatim repeat of this year's inflation-rate into next year — two specifications that often appear in productivity analysts' user-cost constructs — up to an ARMA(2,2) treatment (preferred by only three of the twenty-five types). All the models except the verbatim-repeater allowed for observational error.

Table 2, below, presents a summary of the Kalman filter results. Most assets have fairly long lead times before the 1987 start-date of this paper's Federal Net Operating Surplus estimates; a few (i.e., Defense Vehicles, Non-Defense Commercial and Other Structures) begin right at 1987-88 owing to my truncations of earlier observations in the name of stationarity; one (i.e., Power Structures) begins late. Autoregressive and ARMA specifications dominate; most ARMA models wound up needing moving-average restrictions to preserve invertibility. For three asset-types (i.e., Defense Vehicles and Non-Defense Power Structures and Software), the best forecasts were for no inflation. For every one of the twenty-five assets, the static model of inflation-expectations (i.e., the verbatim repeater) was the least preferred of all ten models. All the selected specifications are noisy: the share of the observational

Table 2: Kalman Filter Summary

Asset Type	Sample Period	Favored Specification	Invertibility Restrictions	Observ'n Var. / Final Innov. Var.
Defense Equipment				
Aircraft	1974-75 – 2018-19	AR(2)		.77
Missiles	1972-73 – 2018-19	ARMA(2,1)	MA1 = .99	.71
Ships	1972-73 – 2018-19	AR(1)		.56
Vehicles	1987-88 – 2018-19	0 inflation		.83
Electronics	1973-74 – 2018-19	ARMA(1,2)	MA1=1.9701, MA2=.99	.33
Other Equipment	1972-73 – 2018-19	ARMA(2,2)		.28
Defense Structures				
Residential	1979-80 – 2018-19	AR(2)		.83
Industrial	1940-41 – 2018-19[10]	AR(2)		.88
Military Facilities	1951-52 – 2018-19	ARMA(1,1)	MA(1) = .99	.67
Defense Intellectual Property Products				
Software	1981-82 – 2018-19	MA(1)		.48
R&D	1941-42 – 2018-19	ARMA(2,2)	MA(2) = .99	.55

[10] Periods 1945-46 – 1948-49, 1964-65, and 1974-75 – 1976-77 were unavailable or excluded as outliers.

Table 2: Kalman Filter Summary (continued)

Asset Type	Sample Period	Favored Specification	Invertibility Restrictions	Observ'n Var. / Final Innov. Var.
Non-Defense Equipment	1975-76 – 2018-19	ARMA(1,1)	MA1 = .99	.35
Non-Defense Structures				
Office	1949-50 – 2018-19 [11]	ARMA(2,2)	MA1=1.9701, MA2 = .99	.45
Commercial	1987-88 – 2018-19 [12]	AR(2)		.73
Health care	1981-82 – 2018-19	AR(1)		.85
Educational	1975-76 – 2018-19	AR(1)		.92
Public Safety	1959-60 – 2018-19	AR(2)		.93
Amusem't & Rec	1980-81 – 2018-19 [13]	AR(2)		.82
Transportation	1980-81 – 2018-19	AR(2)		.80
Power	1990-91 – 2018-19 [14]	0 inflation		.99
Highways & Streets	1933-34 – 2018-19 [15]	AR(2)		.94
Cons & Developm't	1971-72 – 2018-19	AR(2)		.68
Other Structures	1987-88 – 2018-19	AR(2)		.72
Non-Defense Intellectual Property Products				
Software	1982-83 – 2018-19	0 inflation		.41
R&D	1957-58 – 2018-19	AR(1)		.49

...error variance in the (final, and usually converged) value of the innovation variance ranges from 28 to 99 percent, which calls into question the use of the perfect-foresight framework for modeling inflation-expectations in the user-cost.[16]

To construct forecasts, I opted for the Kalman *smoother*, which back-applies an entire sample's information, even to the earliest observation. In principle, for a long-enough run of (stationary) data from which forecasts are harvested from relatively recent observations, this would not be necessary, but for distinctly finite samples, the smoother permits a bit of convergence make-believe; for the final observation the filtered and smoothed results are the same anyway.[17] Below, an extended Figure 1 presents plots of observed and forecast/smoothed inflation-rates for each of the twenty-five assets:

---

[11] Periods 1954-55 – 1955-56 were unavailable or excluded as outliers.

[12] Periods 1993-94 – 1994-95, 2004-05 – 2005-06, and 2016-17 – 2017-18 were unavailable or excluded as outliers.

[13] Period 2006-07 was excluded as an outlier. I probably should have excluded 2012-13, also.

[14] Periods 1992-93 – 1994-95 were unavailable or excluded as outliers.

[15] Periods 1939-40 – 1948-49, 1954-55 – 1955-56, 1963-64, and 1975-76, 1979-80, and 1986-87 were unavailable or excluded as outliers.

[16] Perfect foresight also compels analysts to wait a period for next period's price, while the Kalman filter produces genuine (even optimal) forecasts.

[17] In an agency setting, estimating the filter/smoother's coefficients need only occur every several years in "normal times," permitting smoothing back to the start of the series and then simple filtering/forecasting for subsequent years, until the next comprehensive revision.

Figure 1: Observed *versus* Forecast/Smoothed Asset-Type Inflation-Rates

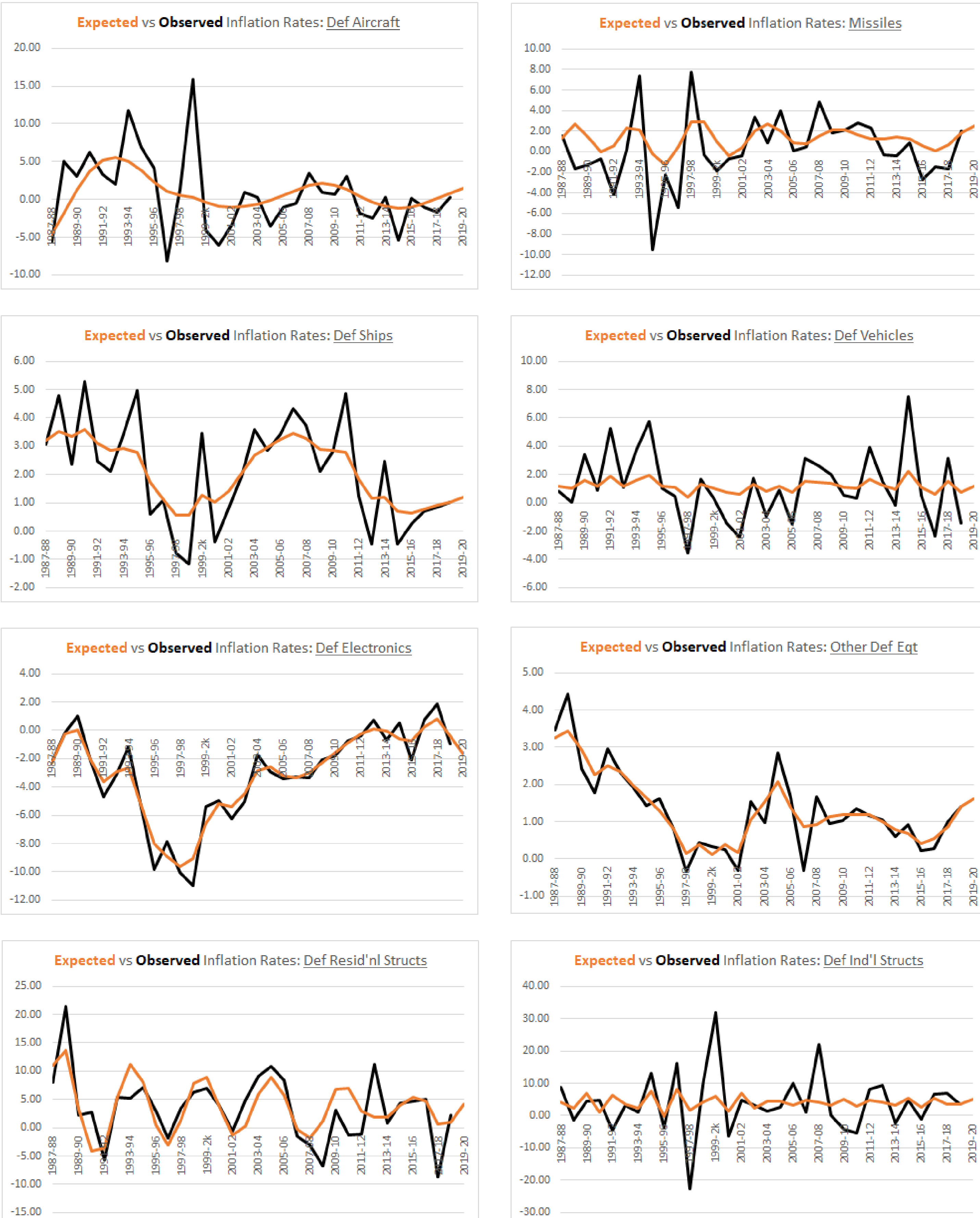
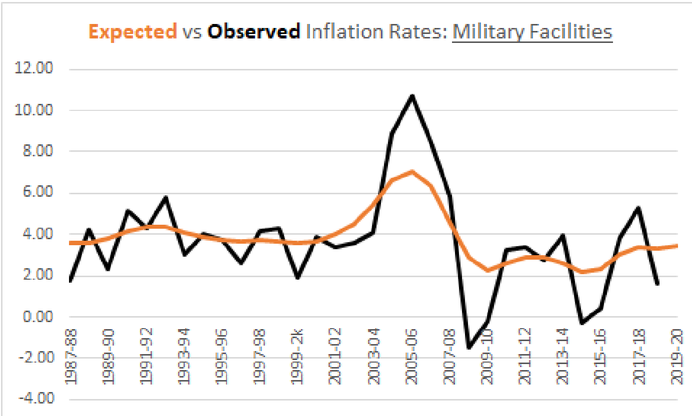
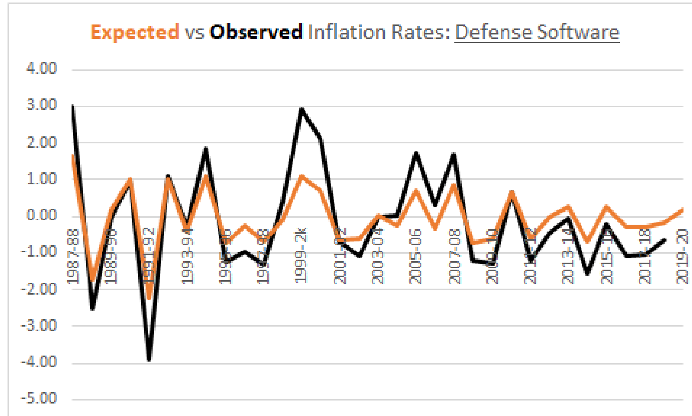
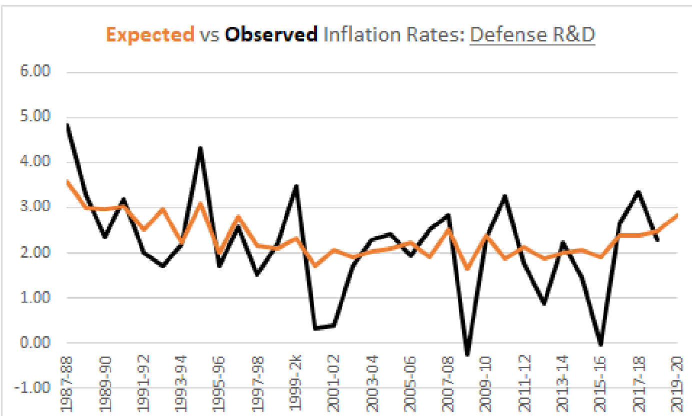
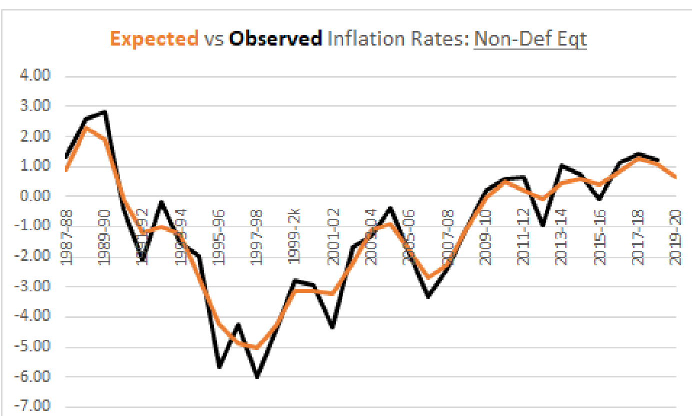


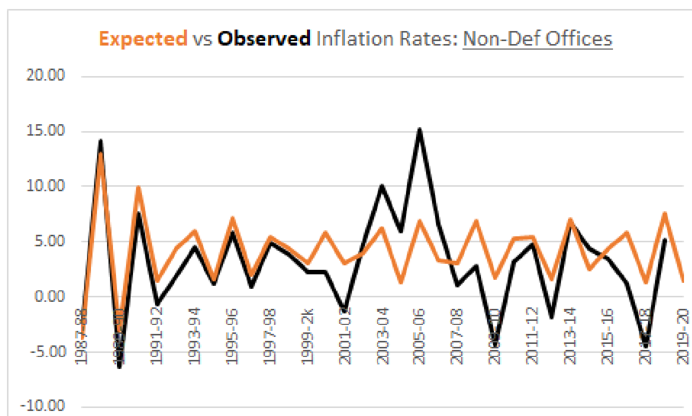
Figure 1: Observed *versus* Forecast/Smoothed Asset-Type Inflation-Rates (continued)

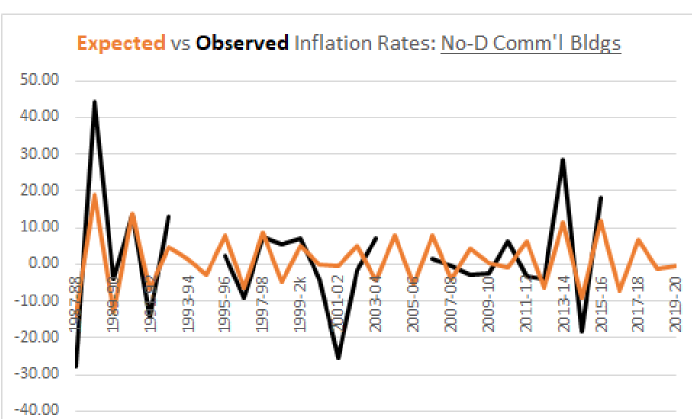












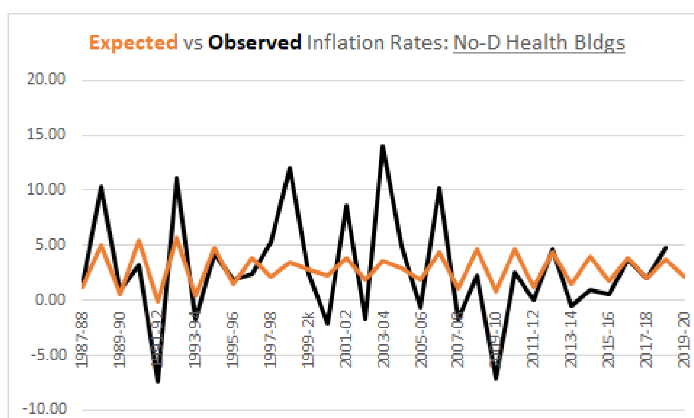


Figure 1: Observed *versus* Forecast/Smoothed Asset-Type Inflation-Rates (continued)

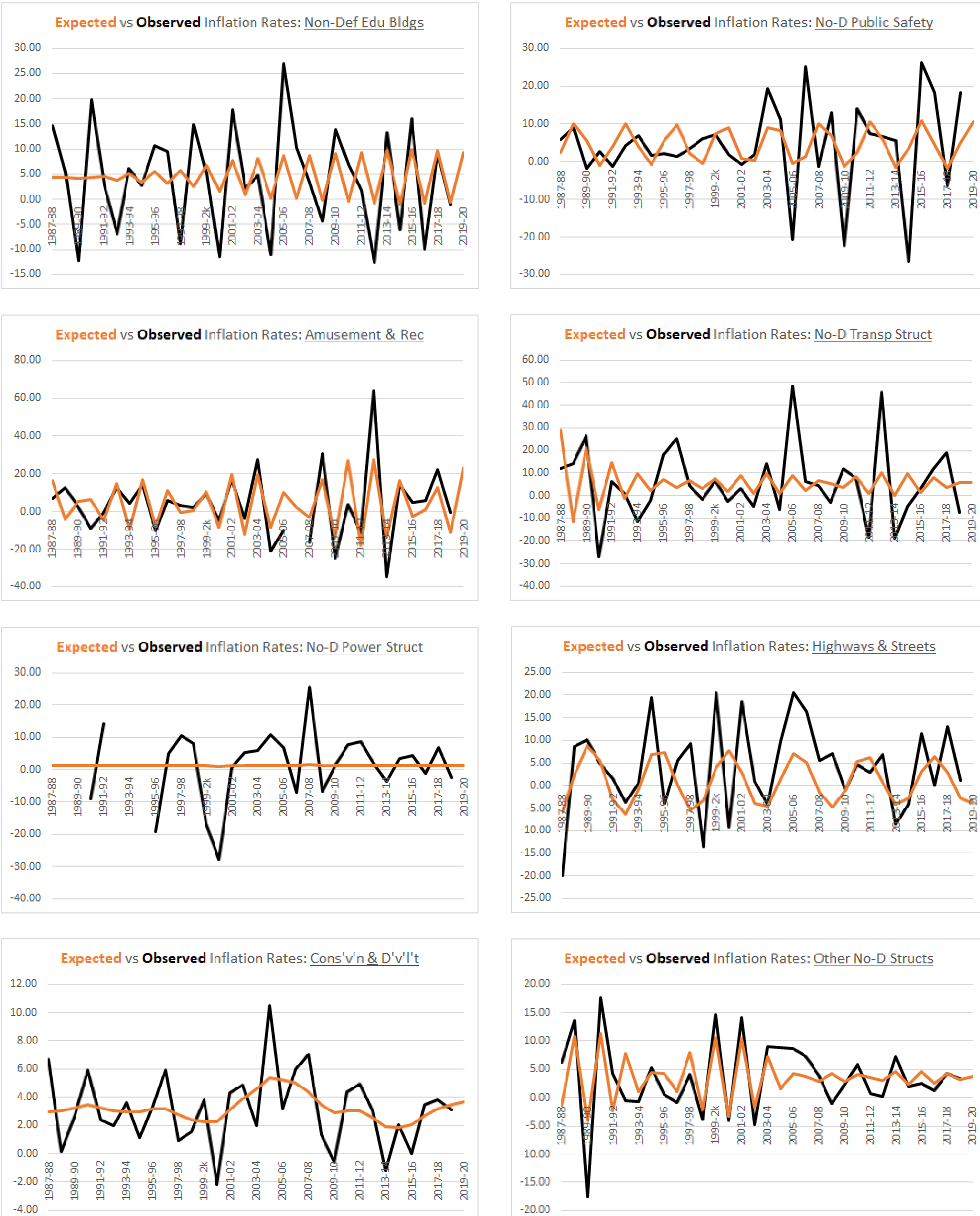
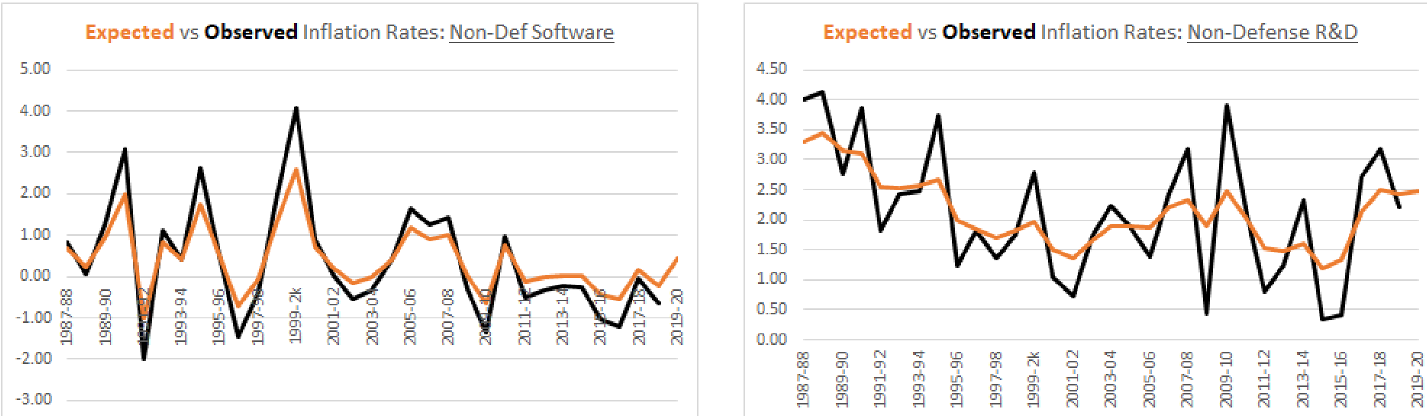


Figure 1: Observed *versus* Forecast/Smoothed Asset-Type Inflation-Rates (continued)



It is apparent the filter/smoother procedure agrees with the observed inflation rates fairly well for most equipment and intellectual-property asset-types, if less well for many structures. The consequences of a poor fit are mild, however: a user-cost with a rather tame revaluation term. Finally, to adapt expected inflation rates from the symmetric log-differenced form that I used as raw material for the Kalman work, to discrete asymmetric forms more in keeping with simple user-cost constructs,[18] i.e.:

$$\mathcal{E}_t\left(\frac{P_{t+1}}{P_t}\right) - 1 \qquad (20)$$

...for each asset-type, I exponentiated the sum of: each Kalman-smoothed *ln*-differenced-type inflation rate, the mean of the originally observed *ln*-differenced prices (which I had removed before beginning the Kalman work), the (small) mean of the differences between the demeaned *ln*-differenced prices and the smoothed estimates, and *half* the variance of those differences.[19] Table 3, below, gives calculated expected inflation-rates per form (20)[20] for all twenty-five asset-types from 1987-88 through 2018-19.[21] Subtracting these from the zero-horizon nominal interest rates of Table 1, above, gives each asset-type's own rate-of-return pattern through the years.

---

[18] The user-cost has several versions. The form $P_t(i_t + \delta - \mathcal{E}_t\hat{P})$, where P_t is the new investment price at date t, i_t is nominal interest rate then, δ is the depreciation rate (often taken as a constant), and $\mathcal{E}_t\hat{P}$ is the expected inflation/revaluation rate given information available then, comports well with the use of the "own rate-of-return" $r_t = i_t - \mathcal{E}_t\hat{P}$, but the form $P_t(i_t + \delta) - \mathcal{E}_t\dot{P}$, where $\mathcal{E}_t\dot{P}$ is the expected price change, has seniority. In a discrete-time setting, with $\dot{P} = P_{t+1} - P_t$, the two forms are reconciled if $\hat{P} = \frac{P_{t+1}}{P_t} - 1$, provided $P_t\left[\mathcal{E}_t\left(\frac{P_{t+1}}{P_t}\right)\text{-}1\right] \approx \mathcal{E}_t(P_{t+1} - P_t)$.

[19] For a Normally distributed random variable x with mean μ and variance σ^2, $\mathcal{E}\, e^x = e^{\mu + ½\sigma^2}$.

[20] ...times 100

[21] The many plots in Figure 1 also follow form (20) type growth rates, not *ln*-differences.

Table 3: Estimates of Expected Investment-Price Revaluation Rates (% change)

Asset Type	1987- -88	1988- -89	1989- -90	1990- -91	1991- -92	1992- -93	1993- -94	1994- -95	1995- -96	1996- -97	1997- -98	1998- -99	1999- -2000	2000- -01	2001- -02	2002- -03	2003- -04
Defense Equipment																	
Aircraft	−4.65	−1.87	1.18	3.69	5.14	5.53	5.04	3.79	2.30	1.12	0.59	0.26	−0.37	−0.90	−1.06	−0.90	−0.58
Missiles	1.36	2.71	1.44	0.01	0.59	2.29	2.12	−0.17	−1.29	0.47	2.89	2.92	0.92	−0.36	0.36	2.01	2.74
Ships	3.19	3.54	3.35	3.58	3.09	2.85	2.94	2.80	1.73	1.14	0.56	0.55	1.26	1.03	1.41	2.03	2.69
Vehicles	1.15	1.00	1.59	1.15	1.89	1.19	1.63	1.97	1.17	1.08	0.38	1.29	1.06	0.76	0.58	1.30	0.84
Electronics	−2.28	−0.23	0.03	−2.06	−3.66	−2.91	−2.66	−5.31	−7.98	−8.93	−9.59	−9.02	−6.56	−5.20	−5.40	−4.46	−2.82
Other Equipment	3.25	3.44	2.93	2.25	2.49	2.33	1.96	1.64	1.30	0.81	0.14	0.38	0.13	0.38	0.16	1.06	1.54
Defense Structures																	
Residential	11.09	13.64	3.32	−4.07	−3.60	5.40	11.19	8.00	0.44	−3.05	1.41	7.93	8.90	3.68	−1.26	0.31	5.91
Industrial	4.06	2.26	7.06	0.99	6.23	3.62	2.43	7.49	−0.12	8.03	1.66	4.26	5.87	1.24	6.82	2.29	4.56
Military Facilities	3.59	3.61	3.82	4.15	4.40	4.36	4.11	3.91	3.76	3.68	3.73	3.68	3.57	3.70	4.00	4.49	5.43
Defense Intellectual Property Products																	
Software	1.64	−1.70	0.19	1.00	−2.22	1.00	−0.40	1.10	−0.73	−0.27	−0.70	−0.11	1.08	0.69	−0.64	−0.61	0.02
R&D	3.58	2.99	2.96	3.04	2.52	2.97	2.22	3.08	2.00	2.81	2.16	2.09	2.31	1.72	2.08	1.91	2.02
Non-Defense Equipment	0.91	2.27	1.90	−0.08	−1.21	−0.98	−1.25	−2.72	−4.27	−4.88	−5.01	−4.28	−3.11	−3.14	−3.24	−2.19	−1.12
Non-Defense Structures																	
Office	−3.62	12.99	−3.04	9.91	1.47	4.44	5.98	1.55	7.15	1.99	5.43	4.33	3.11	5.87	3.08	4.01	6.21
Commercial	−14.01	19.17	−12.97	13.96	−6.79	4.89	1.62	−3.00	7.91	−6.39	8.83	−4.79	4.91	0.00	−0.56	5.02	−4.35
Health care	1.14	5.03	0.56	5.37	−0.05	5.71	0.45	4.72	1.44	3.83	2.13	3.49	2.80	2.20	3.82	1.83	3.63
Educational	4.39	4.43	4.20	4.39	4.63	3.77	5.15	3.40	5.49	3.13	5.79	2.58	6.71	1.52	7.66	0.91	8.22
Public Safety	2.39	9.98	5.63	−0.92	4.04	10.05	3.98	−0.82	5.72	9.75	2.31	−0.40	7.43	9.14	0.74	0.30	9.03
Amusem’t & Rec	16.23	−4.38	5.19	6.17	−4.62	15.02	−9.50	17.22	−7.93	11.27	−0.20	0.79	10.82	−8.50	19.50	−12.33	19.77
Transportation	29.19	−11.67	21.15	−6.12	14.44	−1.20	9.64	2.10	7.19	3.47	6.71	3.13	7.63	1.95	8.94	0.89	9.67
Power	1.29	1.29	1.29	1.21	1.39	1.29	1.29	1.29	1.11	1.32	1.36	1.34	1.14	1.03	1.29	1.32	1.33
Highways & Streets	−5.85	2.22	8.95	5.65	−2.93	−6.24	−0.91	6.85	7.29	0.12	−5.45	−3.27	4.11	7.69	2.96	−3.83	−4.62
Cons & Developm’t	2.96	3.00	3.25	3.47	3.25	3.04	2.99	2.97	3.16	3.18	2.72	2.40	2.28	2.29	3.07	3.89	4.59
Other Structures	−1.39	10.76	−4.09	11.29	−2.20	7.79	1.06	4.49	4.29	1.07	7.89	−2.08	10.62	−3.31	10.37	−1.67	7.30
Non-Defense Intellectual Property Products																	
Software	0.68	0.22	0.93	2.01	−1.01	0.84	0.42	1.74	0.50	−0.70	−0.05	1.36	2.59	0.72	0.20	−0.14	−0.02
R&D	3.31	3.46	3.15	3.11	2.56	2.53	2.57	2.66	2.00	1.85	1.71	1.82	1.96	1.51	1.37	1.66	1.90

Table 3: Estimates of Expected Investment-Price Revaluation Rates (% change, continued)

Asset Type	2004--05	2005--06	2006--07	2007--08	2008--09	2009--10	2010--11	2011--12	2012--13	2013--14	2014--15	2015--16	2016--17	2017--18	2018--19	2019--20
Defense Equipment																
Aircraft	−0.13	0.52	1.24	1.84	2.07	1.87	1.29	0.44	−0.38	−0.95	−1.20	−1.00	−0.52	0.14	0.87	1.48
Missiles	1.99	0.89	0.76	1.55	2.16	2.09	1.65	1.27	1.27	1.47	1.26	0.53	0.10	0.67	1.85	2.49
Ships	2.95	3.24	3.45	3.27	2.89	2.84	2.79	1.85	1.18	1.21	0.69	0.65	0.77	0.90	1.04	1.19
Vehicles	1.15	0.73	1.54	1.45	1.35	1.10	1.06	1.67	1.27	0.97	2.26	1.10	0.59	1.54	0.75	1.20
Electronics	−2.57	−3.20	−3.37	−2.97	−2.30	−1.60	−0.90	−0.24	0.12	−0.03	−0.62	−0.76	0.28	0.82	−0.38	−1.68
Other Equipment	2.06	1.38	0.87	0.92	1.14	1.19	1.19	1.19	1.01	0.80	0.69	0.42	0.55	0.88	1.39	1.62
Defense Structures																
Residential	8.86	5.78	−0.20	−1.81	1.12	6.76	7.01	2.91	1.82	1.82	3.92	5.32	4.65	0.70	1.00	4.16
Industrial	4.53	3.18	4.68	4.07	3.34	5.22	2.84	4.81	4.10	3.16	5.51	2.50	5.24	3.60	3.68	5.10
Military Facilities	6.60	7.08	6.32	4.59	2.86	2.26	2.58	2.88	2.89	2.61	2.21	2.34	3.00	3.38	3.34	3.43
Defense Intellectual Property Products																
Software	−0.27	0.69	−0.33	0.87	−0.73	−0.60	0.66	−0.59	−0.02	0.26	−0.68	0.28	−0.31	−0.30	−0.18	0.19
R&D	2.10	2.22	1.90	2.51	1.66	2.39	1.87	2.13	1.86	1.99	2.07	1.90	2.38	2.40	2.47	2.83
Non-Defense Equipment	−0.92	−1.80	−2.68	−2.26	−1.08	−0.03	0.49	0.21	−0.09	0.44	0.58	0.39	0.85	1.26	1.09	0.66
Non-Defense Structures																
Office	1.31	6.87	3.36	3.02	6.89	1.72	5.27	5.37	1.65	7.07	2.51	4.34	5.88	1.40	7.54	1.53
Commercial	7.92	−5.63	7.71	−3.99	4.39	0.27	−0.99	6.32	−6.56	11.58	−9.23	11.92	−7.03	6.81	−1.07	−0.27
Health care	2.97	1.92	4.38	1.08	4.68	0.76	4.64	1.17	4.36	1.47	3.94	1.77	3.81	1.94	3.68	2.10
Educational	0.32	8.74	0.18	8.83	−0.09	9.14	−0.29	9.40	−0.71	9.80	−0.88	9.88	−0.83	9.64	−0.51	9.30
Public Safety	8.20	−0.56	1.39	10.02	7.07	−1.27	2.43	10.60	5.97	−1.54	3.41	10.95	4.61	−1.67	5.17	10.68
Amusem’t & Rec	−8.62	10.28	2.38	−3.20	16.71	−13.75	27.20	−17.75	27.55	−13.90	16.42	−2.39	0.94	12.49	−10.83	23.32
Transportation	0.82	8.92	2.37	6.50	5.19	3.41	8.27	0.87	10.14	0.21	9.69	1.44	7.78	3.57	5.50	5.66
Power	1.36	1.33	1.22	1.46	1.23	1.29	1.34	1.35	1.30	1.25	1.31	1.32	1.27	1.33	1.26	1.29
Highways & Streets	1.35	7.03	5.16	−1.59	−4.85	−1.10	5.27	6.31	0.75	−4.23	−2.86	3.19	6.54	2.94	−2.84	−3.82
Cons & Developm’t	5.34	5.24	5.00	4.40	3.43	2.90	3.03	3.00	2.51	1.91	1.85	2.07	2.67	3.16	3.46	3.68
Other Structures	1.61	4.23	3.77	2.93	4.28	2.85	4.04	3.53	3.00	4.54	2.42	4.67	2.59	4.28	3.13	3.70
Non-Defense Intellectual Property Products																
Software	0.42	1.17	0.92	1.02	0.01	−0.63	0.77	−0.11	−0.02	0.04	0.03	−0.43	−0.55	0.15	−0.21	0.45
R&D	1.89	1.88	2.20	2.34	1.89	2.47	2.04	1.52	1.48	1.60	1.18	1.34	2.14	2.51	2.42	2.48

5. Asset-Type Depreciation Rates

For deflation-level assets, geometric perpetual-inventory recursions often treat new investment as occurring at the end of the accounting period — i.e., not in place long enough to have suffered any depreciation:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{21}$$

...where K_t is the end-of-year-t real net stock, δ is the depreciation rate, and I_t is the year's real investment flow. Sometimes start-of-year investment is assumed:

$$K_t = (1-\delta)(K_{t-1} + I_t) \tag{22}$$

Either way, stocks and flows are priced using the (new) asset investment deflator, P_{I_t}. The U.S. BEA's "mid-year convention" effectively takes an equal-weighted average of the two views:

$$K_t = (1-\delta)K_{t-1} + (1-\delta/2)I_t \tag{23}$$

...which implies real depreciation flows:

$$D_t = \delta(K_{t-1} + I_t/2) \tag{24}$$

...so that $K_t + ½I_t$ is the relevant stock for service-flow purposes.[22] Nominal consumption of fixed capital attributable to the asset is just $P_{I_t}D_t$, so the asset's implicit depreciation rate is:

$$\delta_t = \frac{nominal\ CFC_t}{P_{I_t}(K_{t-1}+I_t/2)} \tag{25}$$

I used form (25) to derive depreciation rates compatible with a user-cost interpretation of gross operating surplus for each of the twenty-five asset-types owned by the U.S. federal government, despite the fact that most of them are aggregated well above the deflation level.[23] "Nominal CFC" was obtained directly from U.S. Fixed Asset Table 7.3, "Current-Cost Depreciation of Government Fixed Assets." New investment deflators are implicit, the quotients of Fixed Asset Table 7.5, "Investment in Government Fixed Assets" and Table 7.6, "Chain-Type Quantity Indexes for Investment in Government Fixed Assets," and rebased to 1 in 2012. Real net stocks are proportional to entries in Table 7.2, "Chain-Type Quantity Indexes for Net Stock of Government Fixed Assets," rebased to the 2012 values of Table 7.1, "Current-Cost Net Stock of Government Fixed Assets."[24] Real investments are proportional to entries in Table 7.6, rebased to the 2012 values of Table 7.5. Below, Table 4, "Implicit Depreciation Rates," presents the results.

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[22] Further manipulations would show $D_t = \frac{\delta}{1-\delta}(K_t - I_t/2) = \frac{\delta}{1-\delta/2}\left(\frac{K_{t-1}+K_t}{2}\right)$ at the deflation-level.

[23] In this setting, the main problem is that the aggregate implicit deflators for depreciation and investment are bound to differ, owing to the different implied weights in their construction. Forms (21)-(24) are all in volume terms, and (25) would seem to be also, were the investment deflator in the denominator replaced by the depreciation deflator. My use of the investment deflator turns (25) into a ratio of values instead, but is required by the user-cost, which multiplies the sum of the depreciation and own-return rates by the investment price.

[24] This is slightly off, owing to differences in the timing of prices used in the aggregation of deflation-level investment *versus* deflation-level net stocks.

Table 4: Implicit Depreciation Rates (%)

Asset Type	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Defense Equipment																	
Aircraft	7.61	7.66	7.67	7.74	7.63	7.45	7.53	7.23	6.88	6.64	6.93	7.06	6.49	7.10	8.00	8.40	8.26
Missiles	7.79	7.77	7.81	7.91	8.02	8.16	8.31	8.64	9.62	9.75	9.87	10.31	10.39	10.82	11.06	11.16	11.16
Ships	7.86	7.92	8.02	8.03	8.13	8.11	8.10	8.08	8.13	8.08	8.05	8.07	8.08	7.97	8.06	8.07	8.09
Vehicles	12.77	12.31	11.97	11.40	11.90	11.60	11.86	12.28	11.00	10.86	11.04	11.51	11.73	11.46	11.81	12.55	12.42
Electronics	10.88	11.56	11.50	11.29	11.33	11.54	11.78	11.46	11.93	12.79	13.61	14.34	15.45	16.20	16.88	17.22	17.91
Other Equipment	13.26	13.28	13.24	13.23	13.30	13.14	13.19	13.20	13.22	13.30	13.29	13.36	13.30	13.40	13.43	13.54	13.53
Defense Structures																	
Residential	1.36	1.46	1.37	1.31	1.25	1.32	1.42	1.33	1.39	1.34	1.35	1.30	1.35	1.26	1.33	1.34	1.39
Industrial	2.77	2.75	2.83	2.75	2.82	2.98	2.93	2.95	2.66	2.82	2.48	3.48	3.26	2.54	2.77	2.85	2.81
Military Facilities	1.80	1.77	1.77	1.78	1.78	1.79	1.81	1.80	1.80	1.81	1.80	1.80	1.79	1.83	1.83	1.81	1.78
Defense Intellectual Property Products																	
Software	30.47	30.23	30.20	30.37	30.77	31.07	31.67	31.27	31.82	32.16	32.68	33.13	34.21	34.28	33.97	33.98	34.62
R&D	18.90	18.90	18.89	18.88	18.85	18.85	18.84	18.78	18.81	18.74	18.74	18.74	18.74	18.68	18.67	18.62	18.63
Non-Defense Equipment	7.86	8.15	8.18	8.38	8.59	8.72	8.64	8.65	9.04	9.50	9.88	10.47	11.21	11.80	11.77	11.95	11.89
Non-Defense Structures																	
Office	1.68	1.68	1.75	1.84	1.67	1.64	1.82	1.70	1.89	1.74	1.68	1.76	1.84	1.77	1.74	1.93	1.83
Commercial	1.23	1.64	1.14	1.19	1.04	1.20	1.06	1.04[25]	1.04	1.02	1.11	1.35	1.22	1.11	1.18	1.58	1.60
Health care	1.93	1.85	1.64	1.59	1.89	2.00	1.77	1.76	1.99	1.92	1.83	1.71	1.76	1.72	1.76	1.85	1.87
Educational	2.10	1.82	1.72	1.95	1.61	2.06	2.18	2.02	1.94	1.73	1.57	2.17	1.90	1.82	2.06	1.74	1.69
Public Safety	1.78	2.06	1.87	1.87	1.77	2.07	1.90	2.00	1.90	1.80	1.97	1.86	1.92	1.96	1.91	1.91	2.06
Amusem't & Rec	1.82	1.69	1.48	1.43	1.56	1.53	1.33	1.26	1.61	1.75	1.62	1.54	1.48	1.33	1.85	1.58	1.64
Transportation	1.23	1.10	0.96	1.52	2.07	1.94	1.91	2.14	2.17	1.82	1.43	1.35	1.36	1.26	1.27	1.82	1.88
Power	1.52[26]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	1.52[23]	3.16
Highways & Streets	1.51	1.89	1.74	2.37	2.25	2.19	2.25	2.22	1.84	1.89	1.77	1.61	2.46	2.03	2.21	1.85	1.82
Cons & Developm't	1.55	1.45	1.53	1.57	1.48	1.44	1.49	1.44	1.48	1.51	1.46	1.45	1.49	1.45	1.54	1.48	1.47
Other Structures	1.73	1.59	1.37	1.61	1.31	1.18	2.23	2.10	1.88	1.77	1.71	1.59	1.60	2.03	2.05	1.75	1.79
Non-Defense Intellectual Property Products																	
Software	30.83	31.76	31.82	31.38	31.83	31.89	32.33	32.43	32.58	32.44	33.04	33.46	33.99	33.79	33.61	33.63	34.16
R&D	9.38	9.43	9.43	9.41	9.39	9.41	9.43	9.39	9.42	9.45	9.43	9.42	9.45	9.44	9.45	9.45	9.47

[25] No implicit investment deflator was obtainable from Fixed Asset Tables 7.5 and 7.6 for 1994, so I used the geometric mean of the implicit 1993 and 1995 deflators.

[26] No implicit investment deflator was obtainable from Fixed Asset Tables 7.5 and 7.6, for 1988-9 and 1993-4, so I used rolling geometric means of the nearest implicit deflators.

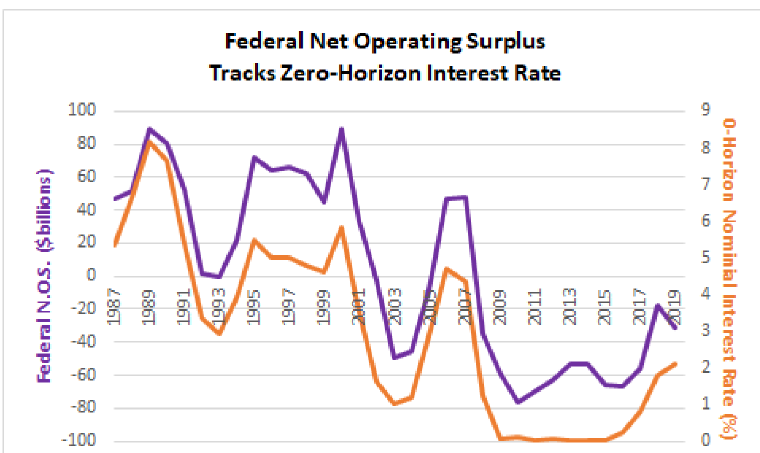
Table 4: Implicit Depreciation Rates (%, continued)

Asset Type	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Defense Equipment																
Aircraft	8.50	8.84	9.08	9.25	9.26	9.27	9.35	9.44	9.58	9.65	9.62	9.66	9.90	9.98	10.34	10.34
Missiles	11.68	11.25	11.36	11.35	11.09	10.64	10.61	10.52	10.20	10.20	10.05	9.97	10.05	10.14	10.47	10.37
Ships	8.13	8.15	8.19	8.24	8.25	8.30	8.29	8.25	8.24	8.14	7.97	7.80	7.68	7.64	7.58	7.50
Vehicles	13.31	14.20	14.78	15.78	16.47	17.30	17.70	17.66	17.53	17.26	17.25	17.47	17.57	17.84	17.74	16.72
Electronics	18.04	18.43	19.02	19.25	19.56	19.54	19.49	19.37	19.40	19.25	19.47	19.07	19.38	19.54	19.44	19.74
Other Equipment	13.60	13.62	13.58	13.65	13.61	13.56	13.52	13.53	13.49	13.48	13.47	13.44	13.45	13.42	13.46	13.46
Defense Structures																
Residential	1.38	1.34	1.32	1.34	1.38	1.38	1.34	1.36	1.37	1.33	1.33	1.28	1.24	1.27	1.40	1.47
Industrial	2.98	3.13	3.05	3.24	2.71	2.64	2.68	2.89	2.86	2.67	2.79	2.72	2.92	2.78	2.77	2.83
Military Facilities	1.80	1.82	1.81	1.79	1.80	1.80	1.80	1.81	1.79	1.79	1.79	1.80	1.83	1.82	1.79	1.80
Defense Intellectual Property Products																
Software	35.07	35.74	35.43	35.85	35.40	36.06	35.45	35.73	35.77	36.19	36.82	37.42	37.20	37.73	38.08	38.48
R&D	18.65	18.67	18.68	18.70	18.69	18.64	18.65	18.60	18.59	18.55	18.50	18.49	18.37	18.34	18.34	18.30
Non-Defense Equipment	11.85	11.78	12.13	12.36	12.46	12.68	12.45	12.32	12.37	12.36	12.20	12.13	12.13	12.09	12.08	12.10
Non-Defense Structures																
Office	1.80	1.83	1.72	1.27	1.81	1.76	1.84	1.76	1.78	1.81	1.70	1.74	1.70	1.70	1.81	1.82
Commercial	1.85	0.90	1.55	0.60	1.78	1.80	1.51	1.67	1.71	1.78	1.40	2.00	1.71	0.96	2.07	2.25
Health care	1.65	1.77	1.99	1.81	1.84	1.79	1.89	1.80	1.75	1.79	1.76	1.87	1.83	1.74	1.82	1.85
Educational	1.93	2.17	2.02	1.84	1.80	2.17	1.92	1.81	1.79	2.07	1.84	1.97	1.72	2.18	2.02	2.06
Public Safety	1.74	1.73	2.41	1.95	2.00	1.79	2.15	1.90	1.78	1.69	1.77	2.44	2.11	1.81	1.96	1.80
Amusem’t & Rec	1.29	1.64	2.29	1.46	1.76	1.35	1.45	1.73	1.95	1.20	1.87	1.65	1.58	1.80	1.48	1.50
Transportation	1.64	1.74	1.18	1.50	1.44	1.49	1.34	1.25	1.93	1.33	1.66	1.76	1.71	1.54	1.30	1.42
Power	2.38	1.84	1.54	1.55	2.32	2.31	2.10	1.79	1.52	2.10	2.06	1.86	1.67	2.17	2.00	2.03
Highways & Streets	1.88	2.13	2.12	1.82	2.01	1.89	1.91	1.82	2.03	1.92	2.11	2.22	2.01	2.01	1.78	1.99
Cons & Developm’t	1.50	1.46	1.52	1.48	1.46	1.48	1.53	1.50	1.50	1.49	1.52	1.53	1.49	1.50	1.50	1.51
Other Structures	1.60	1.89	1.65	1.84	1.71	1.69	1.95	1.80	1.75	1.71	1.83	1.76	1.70	1.67	1.83	1.77
Non-Defense Intellectual Property Products																
Software	34.19	34.59	34.65	34.46	34.64	34.64	34.56	34.43	34.60	34.92	35.05	35.18	35.41	35.69	35.57	35.87
R&D	9.49	9.52	9.56	9.57	9.58	9.60	9.60	9.63	9.64	9.63	9.63	9.63	9.64	9.63	9.64	9.63

6. User-Costs and Gross and Net Operating Surplus

The remainder of the paper assembles the pieces already constructed and considers the results. First, Table 5, below, presents each asset type's year-by-year user-cost as a percent of the type's new investment deflator. User-costs for equipment and intellectual property assets are unfailingly positive, ranging between 5 and 40 percent of their respective investment prices. For structures assets, however, user-costs are negative two-fifths of the time. At its face, the finding flags assets that are not "profitable" for the government to hold, at least temporarily. Yet structures are the most "fixed" of the fixed assets, and even for-profit firms retain money-losing assets for some time so long as revenues cover the assets' associated variable costs. A quick remedy would ignore the Kalman filter/smoother results, set structures' revaluations to zero, and replace $r + \delta$ by $i + \delta$ which is always positive. It may also be that longstanding U.S. estimates of structures' depreciation rates are too low.[27]

Products of own-rates-of-return, new-investment deflators, and service-flow stocks[28] across assets $\times$ years, generate this paper's estimates of the Net Operating Surplus of U.S. federally-owned fixed assets. They are reported in Table 6, farther below. Over two-fifths of the entries are negative, including not a few equipment and intellectual property types. (Adding consumption of fixed capital back in, to yield an enhanced gross operating surplus, would confirm the sign pattern of the user-costs.) Summing across assets year by year gives the entire federal net operating surplus. This was positive for the whole of 1987-2001, averaging about \$52 billion each year, then negative for all but two years of 2002-2019, averaging over –\$36 billion a year. A graph of the federal net operating surplus and the zero-horizon federal bond rate shows tight co-movement between the series. Assets' revaluation rates, which contribute to different surplus patterns type-by-type, roughly net out in this aggregate.



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[27] Or too fixed. For individual-level resale-price profiles of the general form ${}^{P_s}\!/_{P_0} = \left(\frac{e^{\frac{rs}{v-1}} - e^{\frac{rL}{v-1}}}{1 - e^{\frac{rL}{v-1}}}\right)^{v-1}$, $v \geq 2$, which reduce to ${}^{P_s}\!/_{P_0} = \left(1 - \frac{s}{L}\right)^{v-1}$ as r→0 (*cf.* note 5, above), an *extended* Gamma density of service-lives implied by the constraint of a geometric cohort holds approximately unchanged (consistent with the paucity of service-life surveys) despite changes in r *only if* the cohort depreciation-rate δ changes in the opposite direction, particularly when $r < 0$. (v adjusts too.) The upshot is an always-positive user-cost, individual or cohort. (Sliker, 2021).

[28] See equation (24) and the accompanying description, above.

What are we to make of the persistently negative federal NOS. since 2009? Does it reflect governmental inefficiency? Not necessarily. Baumol's Rule predicts the accumulation of unprofitable activities within the taxpayer-supported sector, and certainly the sluggish macroeconomy after 2008 bankrupted many firms, but it is too convenient to suppose such enterprises quickly switched their web-domains from ".com" to ".gov." It is probably more reasonable to consider that many governmental activities, and so the assets that support them, are either foundational in nature (i.e., setting the legal and regulatory stage for observationally productive businesses to flourish) or prudential (i.e., backstopping activities against systemic bad events that insurers would not cover, such as wars, recessions, or climatic disasters). The recessionary backstop is probably most pertinent here, and it does not boil down only to issuing cheques. Negative government "profitability" may in this way be the income-side counterpart of activities that prevent more businesses from failing, and that may enable some of the risk-taking for which private enterprise claims full credit. Further, government- and business-owned assets are imperfect substitutes (though this probably is truer for equipment), and the recessionary timing of negative own-rate signals to sell assets is unfortunate: there are few buyers with the resources and foresight to take advantage of bargain-rate federal cast-offs. (BEA's depreciation rates do not reflect this.) We should be cautious, then, about interpreting government accounting results, such as net operating surplus, through analytical lenses that were tuned for business.[29]

[29] For optimal choices of flexible inputs, the relevant guide would not be Hotelling's lemma (which presumes profit maximization) or even Shephard's lemma (which presumes cost minimization but needs a measured output), but Roy's identity (which presumes budget-constrained goal maximization, even though the goals may change every several years), removed from the household setting.

Table 5: User-Costs as Percentages of Implicit Investment Deflators

$100(r + \delta)$

Asset Type	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Defense Equipment																	
Aircraft	17.60	16.19	14.65	11.69	7.87	5.28	5.40	7.36	10.08	10.52	11.35	11.59	11.46	13.82	12.59	10.93	9.86
Missiles	11.77	11.72	14.53	15.54	12.81	9.24	9.10	12.72	16.41	14.28	11.99	12.18	14.07	17.00	14.23	10.77	9.44
Ships	10.02	11.03	12.84	12.09	10.42	8.62	8.08	9.19	11.90	11.94	12.50	12.32	11.43	12.77	10.18	7.66	6.42
Vehicles	16.97	17.96	18.55	17.89	15.39	13.78	13.15	14.22	15.32	14.78	15.68	15.01	15.28	16.53	14.75	12.88	12.60
Electronics	18.50	18.45	19.64	20.99	20.37	17.81	17.35	20.68	25.41	26.73	28.22	28.16	26.62	27.22	25.81	23.31	21.75
Other Equipment	15.35	16.50	18.48	18.62	16.20	14.18	14.15	15.47	17.42	17.49	18.17	17.78	17.77	18.84	16.79	14.11	13.00
Defense Structures																	
Residential	−4.38	−5.53	6.21	13.01	10.24	−0.71	−6.86	−2.75	6.44	9.38	4.96	−1.84	−2.94	3.40	6.12	2.66	−3.50
Industrial	4.05	7.15	3.94	9.39	1.97	2.73	3.42	−0.62	8.28	−0.20	5.83	4.01	2.01	7.12	−0.53	2.19	−0.73
Military Facilities	3.56	4.81	6.12	5.27	2.77	0.80	0.61	1.81	3.54	3.14	3.08	2.91	2.83	3.96	1.36	−1.06	−2.63
Defense Intellectual Property Products																	
Software	34.17	38.58	38.18	37.01	38.37	33.44	34.98	34.09	38.06	37.43	38.40	38.04	37.74	39.41	38.14	36.22	35.62
R&D	20.67	22.57	24.10	23.48	21.72	19.25	19.54	19.61	22.31	20.94	21.59	21.45	21.05	22.78	20.12	18.34	17.63
Non-Defense Equipment	12.29	12.54	14.45	16.09	15.19	13.06	12.81	15.28	18.81	19.38	19.90	19.55	18.93	20.77	18.54	15.77	14.03
Non-Defense Structures																	
Office	10.64	−4.66	12.96	−0.43	5.59	0.56	−1.25	4.06	0.24	4.75	1.27	2.23	3.33	1.73	2.18	−0.45	−3.36
Commercial	20.59	−10.88	22.28	−5.13	13.22	−0.32	2.35	7.95	−1.37	12.41	−2.71	10.93	0.92	6.94	5.26	−1.81	6.97
Health care	6.13	3.48	9.25	3.85	7.33	−0.34	4.23	0.95	6.05	3.09	4.71	3.01	3.57	5.34	1.46	1.64	−0.73
Educational	3.04	4.04	5.69	5.19	2.37	1.66	−0.06	2.53	1.94	3.59	0.80	4.38	−0.20	6.12	−2.07	2.46	−5.51
Public Safety	4.74	−1.27	4.41	10.42	3.12	−4.61	0.83	6.73	1.67	−2.95	4.68	7.05	−0.90	−1.36	4.70	3.24	−5.94
Amusem’t & Rec	−9.06	12.73	4.46	2.90	11.56	−10.12	13.75	−12.05	15.04	−4.52	6.83	5.55	−4.72	15.66	−14.12	15.53	−17.11
Transportation	−22.61	19.43	−12.02	15.27	−6.98	6.51	−4.82	3.95	0.49	3.35	−0.27	3.01	−1.67	5.13	−4.14	2.56	−6.77
Power	5.57	6.89	8.40	7.95	5.52	3.60	3.14	4.15	5.91	5.20	5.17	4.97	4.99	6.32	3.76	1.82	2.85
Highways & Streets	12.70	6.33	0.96	4.36	10.56	11.80	6.08	−0.72	0.05	6.77	12.24	9.68	2.96	0.16	2.78	7.31	7.46
Cons & Developm’t	3.93	5.11	6.45	5.74	3.61	1.77	1.41	2.38	3.82	3.34	3.75	3.85	3.82	4.98	2.00	−0.78	−2.10
Other Structures	8.46	−2.51	13.63	−2.04	8.90	−3.24	4.08	1.52	3.10	5.70	−1.16	8.46	−4.41	11.17	−4.79	5.05	−4.49
Non-Defense Intellectual Property Products																	
Software	35.50	38.20	39.06	37.00	38.22	34.41	34.82	34.60	37.58	38.14	38.10	36.89	36.01	38.89	36.94	35.40	35.19
R&D	11.41	12.63	14.44	13.93	12.21	10.24	9.77	10.64	12.93	12.59	12.73	12.40	12.09	13.75	11.61	9.42	8.59

Table 5: User-Costs as Percentages of Implicit Investment Deflators (continued)

$100(r + \delta)$

Asset Type	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Defense Equipment																
Aircraft	9.83	11.20	12.53	11.78	8.44	7.47	8.16	9.03	10.02	10.64	10.84	10.69	10.65	10.64	11.25	10.98
Missiles	10.90	13.25	15.29	14.17	10.18	8.62	9.07	9.28	8.99	8.77	8.82	9.47	10.18	10.28	10.40	9.99
Ships	6.38	7.79	9.44	9.33	6.61	5.54	5.60	6.43	7.12	6.97	7.30	7.18	7.14	7.54	8.31	8.43
Vehicles	13.36	16.35	17.94	18.69	16.37	16.27	16.74	16.03	16.33	16.34	15.02	16.40	17.21	17.10	18.77	17.63
Electronics	21.82	24.51	27.08	26.59	23.11	21.21	20.49	19.65	19.34	19.32	20.12	19.87	19.32	19.52	21.59	23.54
Other Equipment	12.74	15.13	17.41	17.09	13.72	12.45	12.44	12.38	12.55	12.73	12.80	13.05	13.12	13.33	13.85	13.95
Defense Structures																
Residential	−6.27	−1.56	6.22	7.52	1.51	−5.31	−5.56	−1.52	−0.38	−0.45	−2.57	−4.00	−3.19	1.37	2.18	−0.58
Industrial	−0.34	2.83	3.07	3.54	0.62	−2.50	−0.06	−1.89	−1.18	−0.46	−2.69	0.25	−2.09	−0.01	0.87	−0.16
Military Facilities	−3.60	−2.37	0.18	1.57	0.19	−0.39	−0.68	−1.03	−1.04	−0.78	−0.40	−0.51	−0.95	−0.76	0.23	0.49
Defense Intellectual Property Products																
Software	36.55	37.93	40.45	39.35	37.38	36.74	34.89	36.36	35.85	35.96	37.53	37.17	37.73	38.83	40.04	40.41
R&D	17.75	19.33	21.47	20.56	18.28	16.32	16.89	16.50	16.79	16.59	16.46	16.61	16.21	16.74	17.65	17.58
Non-Defense Equipment	13.98	16.47	19.50	18.99	14.79	12.79	12.06	12.14	12.52	11.97	11.64	11.78	11.50	11.63	12.76	13.55
Non-Defense Structures																
Office	1.69	−2.15	3.06	2.61	−3.83	0.11	−3.33	−3.57	0.19	−5.22	−0.78	−2.57	−3.96	1.11	−3.96	2.41
Commercial	−4.86	9.42	−1.46	8.96	−1.36	1.60	2.60	−4.62	8.34	−9.76	10.65	−9.89	8.97	−5.04	4.92	4.64
Health care	−0.11	2.74	2.31	5.10	−1.58	1.11	−2.65	0.67	−2.55	0.36	−2.15	0.12	−1.76	0.60	−0.08	1.87
Educational	2.81	−3.68	6.53	−2.61	3.14	−6.90	2.31	−7.55	2.56	−7.68	2.74	−7.87	2.78	−6.67	4.31	−5.12
Public Safety	−5.25	5.18	5.71	−3.71	−3.82	3.12	−0.17	−8.67	−4.12	3.27	−1.62	−8.47	−2.28	4.28	−1.44	−6.77
Amusem’t & Rec	11.11	−5.75	4.61	9.03	−13.71	15.17	−25.65	19.51	−25.53	15.15	−14.52	4.07	0.87	−9.88	14.09	−19.70
Transportation	2.02	−4.29	3.51	−0.63	−2.50	−1.84	−6.83	0.42	−8.15	1.17	−8.01	0.35	−5.84	−1.23	−2.42	−2.12
Power	2.23	3.40	5.01	4.45	2.35	1.09	0.86	0.48	0.29	0.89	0.78	0.57	0.62	1.63	2.51	2.86
Highways & Streets	1.73	−2.01	1.65	7.78	8.11	3.06	−3.26	−4.45	1.34	6.19	5.00	−0.94	−4.31	−0.13	6.40	7.93
Cons & Developm’t	−2.63	−0.89	1.21	1.44	−0.72	−1.34	−1.40	−1.47	−0.94	−0.37	−0.30	−0.51	−0.95	−0.86	−0.18	−0.06
Other Structures	1.19	0.55	2.58	3.27	−1.32	−1.08	−1.99	−1.69	−1.19	−2.79	−0.56	−2.87	−0.67	−1.81	0.48	0.19
Non-Defense Intellectual Property Products																
Software	34.97	36.31	38.42	37.80	35.88	35.34	33.89	34.58	34.68	34.92	35.04	35.64	36.18	36.34	37.56	37.54
R&D	8.80	10.53	12.06	11.60	8.94	7.20	7.66	8.15	8.22	8.08	8.47	8.32	7.73	7.92	9.00	9.27

Table 6: Net Operating Surplus of Federally-Owned Fixed Assets
$billions

Asset Type	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Defense Equipment																	
Aircraft	13.93	11.91	10.47	6.12	0.40	−3.52	−3.42	0.22	5.87	7.20	7.28	7.20	8.75	10.88	6.77	3.52	2.21
Missiles	2.66	2.85	5.00	5.97	4.01	0.91	0.69	3.83	5.57	3.49	1.47	1.31	2.41	3.72	1.78	−0.20	−0.89
Ships	1.84	2.80	4.63	4.09	2.49	0.57	−0.03	1.31	4.59	4.63	5.26	4.84	3.69	5.35	2.31	−0.44	−1.86
Vehicles	0.89	1.29	1.54	1.59	0.88	0.58	0.34	0.49	1.06	0.90	1.01	0.70	0.70	0.97	0.55	0.06	0.04
Electronics	2.38	2.32	2.90	3.61	3.35	2.23	1.94	3.22	4.41	4.03	3.86	3.28	2.39	2.31	1.85	1.24	0.79
Other Equipment	1.20	2.01	3.60	3.99	2.31	0.90	0.87	2.12	4.00	4.13	4.92	4.46	4.58	5.65	3.53	0.61	−0.60
Defense Structures																	
Residential	−2.11	−2.87	2.48	6.26	5.02	−1.08	−4.67	−2.45	3.28	5.42	2.40	−2.18	−3.18	1.70	3.94	1.08	−4.22
Industrial	0.70	2.56	0.63	3.87	−0.51	−0.15	0.28	−2.06	3.58	−1.82	2.30	0.27	−0.69	3.25	−2.14	−0.44	−2.40
Military Facilities	4.09	7.25	10.78	8.82	2.62	−2.72	−3.46	0.02	5.30	4.18	4.12	3.70	3.59	7.45	−1.69	−10.61	−16.85
Defense Intellectual Property Products																	
Software	0.33	0.88	0.95	0.90	1.14	0.37	0.54	0.47	1.06	0.88	0.96	0.83	0.61	0.91	0.75	0.39	0.17
R&D	3.05	7.06	10.69	9.79	6.27	0.86	1.49	1.76	7.46	4.60	5.96	5.58	4.71	8.49	2.95	−0.58	−2.15
Non-Defense Equipment	2.20	2.32	3.60	4.88	4.46	3.09	3.18	5.21	7.68	7.59	7.71	6.85	5.85	6.91	5.18	2.88	1.65
Non-Defense Structures																	
Office	2.67	−1.88	3.84	−0.74	1.40	−0.39	−1.18	0.97	−0.70	1.38	−0.20	0.24	0.82	−0.03	0.25	−1.36	−3.12
Commercial	4.70	−2.30	5.54	−1.59	3.51	−0.38	0.37	1.99	−0.70	3.36	−1.03	2.85	−0.10	2.09	1.39	−0.86	1.35
Health care	0.87	0.35	1.86	0.57	1.44	−0.58	0.70	−0.23	1.22	0.37	0.95	0.46	0.72	1.48	−0.12	−0.09	−1.11
Educational	0.14	0.37	0.69	0.50	0.14	−0.08	−0.41	0.10	0.00	0.43	−0.20	0.51	−0.55	1.18	−1.00	0.21	−2.13
Public Safety	0.66	−0.81	0.68	2.29	0.38	−1.93	−0.34	1.66	−0.08	−1.84	1.09	2.24	−1.32	−1.69	1.46	0.69	−4.27
Amusem’t & Rec	−1.19	1.30	0.40	0.20	1.29	−1.52	1.86	−2.12	2.50	−1.07	0.97	0.78	−1.26	3.23	−3.46	3.54	−4.58
Transportation	−1.93	1.66	−1.35	1.82	−0.87	0.47	−0.70	0.17	−0.16	0.17	−0.24	0.25	−0.45	0.62	−0.85	0.12	−1.38
Power	0.08	0.11	0.16	0.17	0.10	0.06	0.05	0.08	0.14	0.10	0.11	0.11	0.12	0.14	0.05	0.01	−0.01
Highways & Streets	1.48	0.47	−0.09	0.25	1.11	1.32	0.51	−0.40	−0.29	0.77	1.77	1.51	0.08	−0.37	0.10	1.18	1.24
Cons & Developm’t	2.62	4.29	5.79	5.05	2.74	0.42	−0.11	1.31	3.32	2.77	3.77	3.96	3.89	6.11	0.78	−3.97	−6.56
Other Structures	0.39	−0.26	0.90	−0.23	0.58	−0.37	0.17	−0.06	0.13	0.44	−0.34	0.87	−0.75	1.35	−1.00	0.57	−1.05
Non-Defense Intellectual Property Products																	
Software	0.59	0.89	1.09	0.93	1.16	0.47	0.49	0.44	1.06	1.23	1.10	0.79	0.52	1.48	1.05	0.58	0.34
R&D	4.41	7.28	12.08	11.40	7.56	2.33	1.01	3.82	11.34	10.50	11.38	10.55	9.69	16.53	8.53	−0.14	−3.76
Totals	**46.65**	**51.85**	**88.86**	**80.51**	**52.98**	**1.86**	**0.17**	**21.87**	**71.64**	**63.84**	**66.38**	**61.96**	**44.82**	**89.71**	**32.96**	**−2.01**	**−49.15**

Table 6: Net Operating Surplus of Federally-Owned Fixed Assets (continued)
$billions

Asset Type	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Defense Equipment																
Aircraft	1.82	3.14	4.60	3.38	−1.14	−2.54	−1.72	−0.62	0.70	1.57	1.99	1.61	1.16	1.03	1.44	1.04
Missiles	−0.39	0.99	1.90	1.34	−0.44	−0.99	−0.77	−0.63	−0.64	−0.77	−0.67	−0.28	0.07	0.07	−0.04	−0.20
Ships	−2.02	−0.42	1.54	1.41	−2.19	−3.77	−3.76	−2.67	−1.66	−1.72	−1.01	−0.94	−0.84	−0.16	1.19	1.56
Vehicles	0.01	0.44	0.68	0.72	−0.03	−0.35	−0.36	−0.65	−0.49	−0.36	−0.79	−0.37	−0.11	−0.21	0.28	0.24
Electronics	0.84	1.45	2.12	2.21	1.25	0.68	0.46	0.13	−0.03	0.03	0.29	0.34	−0.02	−0.01	0.84	1.46
Other Equipment	−1.02	1.96	5.30	4.99	0.17	−1.90	−1.98	−2.24	−1.90	−1.55	−1.37	−0.80	−0.67	−0.17	0.84	1.11
Defense Structures																
Residential	−7.22	−3.03	5.56	6.91	0.14	−6.79	−7.22	−2.97	−1.79	−2.01	−4.40	−6.18	−5.37	0.13	0.89	−2.37
Industrial	−2.23	−0.20	0.01	0.21	−1.78	−4.28	−2.14	−3.47	−3.11	−2.58	−4.32	−1.99	−3.95	−2.31	−1.65	−2.64
Military Facilities	−21.31	−17.94	−7.66	−1.12	−8.69	−11.70	−13.35	−15.87	−16.27	−15.08	−13.20	−13.75	−16.40	−15.62	−9.87	−8.24
Defense Intellectual Property Products																
Software	0.24	0.36	0.85	0.61	0.36	0.12	−0.10	0.12	0.02	−0.05	0.16	−0.06	0.13	0.28	0.52	0.55
R&D	−2.07	1.64	7.36	5.21	−1.23	−7.08	−5.59	−6.91	−6.04	−6.56	−6.88	−6.27	−7.04	−5.22	−2.27	−2.40
Non-Defense Equipment	1.71	3.98	6.62	6.17	2.24	0.11	−0.41	−0.20	0.18	−0.47	−0.68	−0.45	−0.82	−0.62	0.96	2.11
Non-Defense Structures																
Office	−0.07	−2.83	1.08	1.17	−4.98	−1.50	−4.50	−4.83	−1.52	−6.61	−2.48	−4.46	−5.99	−0.63	−5.74	0.61
Commercial	−1.81	4.71	−0.97	2.77	−1.06	−0.07	0.36	−2.26	2.32	−3.89	3.98	−4.16	2.98	−4.37	0.96	0.85
Health care	−0.86	0.49	0.16	1.81	−1.86	−0.38	−2.40	−0.63	−2.46	−0.88	−2.45	−1.12	−2.35	−0.79	−1.36	0.01
Educational	0.27	−1.62	1.57	−1.69	0.52	−3.34	0.16	−4.14	0.34	−3.77	0.39	−3.99	0.49	−3.66	1.02	−3.13
Public Safety	−4.42	2.39	1.78	−3.77	−3.79	0.97	−1.30	−6.67	−3.97	1.12	−2.49	−5.81	−2.91	1.91	−2.43	−7.15
Amusem't & Rec	3.05	−1.80	0.50	2.60	−4.40	5.10	−7.49	5.13	−7.04	5.80	−4.39	0.74	−0.23	−3.89	5.11	−8.50
Transportation	0.07	−1.04	0.59	−0.57	−1.10	−0.89	−2.44	−0.27	−2.61	−0.06	−2.92	−0.40	−2.20	−0.90	−1.43	−1.25
Power	−0.01	0.08	0.23	0.19	0.00	−0.11	−0.12	−0.15	−0.16	−0.17	−0.19	−0.21	−0.19	−0.10	0.10	0.16
Highways & Streets	−0.03	−0.97	−0.13	1.96	2.12	0.44	−1.90	−2.41	−0.27	1.78	1.10	−1.14	−2.52	−0.85	2.07	2.69
Cons & Developm't	−7.69	−4.83	−0.65	−0.08	−5.22	−6.85	−7.08	−7.52	−6.52	−5.12	−4.91	−5.60	−6.71	−6.78	−5.05	−4.88
Other Structures	−0.08	−0.28	0.22	0.39	−0.89	−0.82	−1.21	−1.17	−1.01	−1.58	−0.92	−1.84	−0.97	−1.46	−0.59	−0.72
Non-Defense Intellectual Property Products																
Software	0.25	0.57	1.32	1.23	0.49	0.28	−0.28	0.06	0.04	0.00	0.00	0.25	0.44	0.39	1.26	1.13
R&D	−3.08	4.73	12.11	10.27	−3.41	−13.03	−11.21	−8.93	−8.76	−9.89	−7.59	−8.70	−12.95	−12.13	−4.76	−2.79
Totals	**−46.05**	**−8.03**	**46.69**	**48.32**	**−34.92**	**−58.69**	**−76.35**	**−69.77**	**−62.65**	**−52.82**	**−53.75**	**−65.58**	**−66.97**	**−56.07**	**−17.71**	**−30.75**

7. Further Thoughts

An earlier version of this paper included estimates of Net Operating Surplus attributable to fixed assets owned by states and localities. I have not retained them, as such polities' yield curves might not coincide with the federal yield curve, and the question of whether the federal government would underwrite bankrupt states' debts (which would permit the use of the federal yield curve across the board) is one of the great unknowns of American politics. State and local holdings are even more heavily tipped toward structures than federal assets, so the previous pattern — of net operating surplus swinging positive when the zero-horizon nominal interest rate is high enough, but negative in liquidity-trap conditions — would be amplified.

This essay has shown that the inclusion of a non-zero net operating surplus for government-owned fixed assets is necessary under reasonable assumptions on government discounting: otherwise the dual relations between asset purchase and rental values break down. It has also shown that the data to estimate central government net operating surplus are already at hand and presented results on that surplus (or deficit) that are noticeably sensitive to macroeconomic conditions. A similar exercise for privately-held assets, comparing a user-cost -based net surplus to what is actually computed from companies' books, would be of interest and might usefully inform public policy, although zero-horizon nominal interest rates for private industries would be harder to estimate. What the paper has not, and cannot, show is that even passably optimizing governments must "run like a business."

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