Inequality in Multidimensional Well-Being in the United States

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Paper prepared for the 37th IARIW General Conference  
August 22-26, 2022  
Session 4C-1, New Measures of Global Comparisons in Well-Being and Sustainability III  
Time: Wednesday, August 24, 2022 [14:00-15:30 CEST]
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Abstract

In this paper we provide a framework to measure an individual’s multidimensional well-being and discuss two approaches to measure inequality in multidimensional well-being. The framework is used to study inequality in multidimensional well-being in the United States in the last decade. Using data from the Current Population Survey on three well-being indicators, namely, income, health, and education, we compute a multidimensional well-being index for every individual in the sample. We find that inequality in well-being increased between 2010 and 2014 and decreased between 2014 and 2019. We test the sensitivity of our results by using alternative measures of inequality and attaching alternative weights to well-being indicators.

Keywords: multidimensional well-being, inequality, United States

JEL classification codes: D39, D63, I31
1. INTRODUCTION

There is a long tradition of economists studying income inequality. Though distribution of income in itself may be of interest, income may not always reflect an individual’s well-being. An individual's well-being depends on a variety of factors related to health, employment, housing quality, education, and the environment, the distribution of which may not be proxied by income distribution. Thus, a multidimensional approach to measuring well-being has increasingly gained grounds. Starting in 2010, the United Nations has annually published estimates of a multidimensional poverty index for countries. Since then, multidimensional poverty indices have been in the spotlight and are now estimated by the national statistical offices in several countries. Relatively less attention has been paid to analyzing multidimensional inequality. Measuring multidimensional inequality faces both theoretical and empirical challenges. For example, ranking of individuals on the basis of income is straightforward, but how do we rank individuals on the basis of educational attainment, self-assessed health status, housing quality, and so on? How do we compare distributions of multiple attributes? Do we consider the distribution of each attribute separately or do we combine these attributes in an index first and then examine the distribution of that index? Fortunately, there is a robust and growing literature, which addresses many of these questions.\(^1\)


Among the aggregative strategies is the two-step approach proposed by Maasoumi (1986). The first step is to aggregate for each individual the multiple dimensions of well-being into a single composite index. Maasoumi (1986) used a constant elasticity of substitution utility function to aggregate an individual’s achievements along the different dimensions of well-being. In the second step, some univariate inequality index is used to analyze the distribution of multidimensional well-being. A few studies have used the two-step approach with individual level data to measure inequality in multidimensional well-being (see Section 3 for examples).

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2 Brandolini (2008) also discussed vector dominance and multivariate statistical techniques under the non-aggregative strategies. Among aggregative strategies, an alternative strategy is to measure well-being in terms of equivalent income (see Decancq et al. (2015), Rhode and Guest (2018) for applications). Decancq et al. (2017) allowed for heterogeneity in individual preferences and compute inequality in the distribution of equivalent incomes in Russia for the period 1995-2005.
In this paper, we use this two-step approach to study the change in multidimensional inequality in the United States (U.S.) over the period 2010 - 2019. We first measure multidimensional well-being for each individual and then measure inequality in the well-being index. We consider three attributes, namely, income, education and health status. We compute an individual’s overall achievement over these three attributes, the individual’s overall achievement being defined as a weighted average of her achievements in terms of the different attributes. Next, we convert the overall achievement of each individual to an index of her well-being by a power function and then examine how the distribution of the multidimensional well-being index changed between 2010, 2014 and 2019. We use inequality indices such as the Gini index, the Theil index and the Atkinson index to measure inequality in the multidimensional well-being index.

There are two broad approaches used to combine multiple attributes. In one of these approaches, which we call the Type I approach (or the “column-first” approach), one first considers the individuals’ achievements along each specific dimension of well-being and measures inequality in those dimension-specific achievements. Next, one takes the set of inequality indices along the different dimensions and aggregates them in some fashion in order to arrive at an overall measure of inequality in the society. An example of this approach is the inequality-adjusted Human Development Index (also see Gajdos and Weymark (2005)). A second approach, which we call the Type II approach (or the “row-first” approach), first aggregates an individual’s achievements along all dimensions to assess the overall well-being of that individual and then uses some inequality index to measure inequality in the individuals’
overall well-being levels (see Maasoumi (1986, 1999), Decancq et al. (2009), Decancq and Lugo (2012), Bosmans et al. (2015), and Seth (2013)).

We show both analytically and empirically that the Type I and Type II approaches can provide very different results when measuring inequality in a multidimensional framework (see Sections 2.3 and 4.1 respectively). The difference arises because the Type I approach ignores the impact of the association between different dimensions on inequality of well-being and does not capture the overlap of achievements in different dimensions such as income, health and education, experienced by an individual. The problem is analogous to the problems discussed by Pattanaik et al. (2012) and Pattanaik and Xu (2018) in the context of the measurement of well-being and deprivation in a multidimensional framework. This is not to deny that the assessment of inequality of the individuals’ achievements along each separate dimension can give us very useful information. In fact, in Section 4, we examine information on dimensional inequalities to gain some insight into the results regarding the change in well-being inequality in the United States that we derive using the Type II approach.

The plan of the paper is as follows. In Section 2, we outline the conceptual framework that we use and explain why we chose to use the Type II approach rather than the Type I approach to measuring inequality. In Section 3, we detail the data we compile on multiple dimensions of well-being. In Section 4, we present estimates of well-being inequality in the United States in the last decade. Section 5 concludes.
2. THE CONCEPTUAL FRAMEWORK

2.1. The notation

Let \( F = \{f_1, f_2, \ldots, f_m\} \) \((m \geq 2)\) be the finite set of functionings or attributes which people value in their lives and which constitute the dimensions of individual well-being.\(^3\) Let \( M = \{1, 2, \ldots, m\} \). Attributes can take cardinal values (income) or ordinal values (health status or educational attainment). For every \( j \in M \), let \( V_j \) be the (finite) set of discrete values that \( f_j \) can take, the number of elements in \( V_j \) being denoted by \( r(j) \geq 2 \).\(^4\) Let \( V \) denote \( \times_{j \in M} V_j \). Thus, if \( f_j \) denotes the attribute of health, then \( f_j \) may take values such as poor health, fair health, good health, etc. Let \( A_{n \times m} = (a_{ij})_{n \times m} \) be an achievement matrix with \( a_{ij} \) denoting individual \( i \)'s achievement in dimension \( f_j \). We use \( a_{i\ast} = (a_{i1}, \ldots, a_{im}) \) to denote individual \( i \)'s achievement vector along all the dimensions and \( a_{\ast j} = (a_{1j}, \ldots, a_{nj}) \) to denote the achievement vector of all the individuals along the dimension \( f_j \).

For each \( j \in M \), there is a linear ordering \([\geq j]\) ("at least as high as") defined over \( V_j \), with \([> j]\) ("higher than") being the asymmetric factor of \([\geq j]\). For every \( j \in M \) and every \( v_j \in V_j \), let \( s(v_j) \equiv \#\{v'_j \in V_j: v_j[> j]v'_j\}/(r(j) - 1) \). Thus, the numbers in \( \{s(v_j): v_j \in V_j\} \) are derived by taking the usual "Borda scores", defined in terms of the linear ordering \([\geq j]\), of the different

\(^3\) Sen (1987) defines "functionings" as the "doings" and "beings" that people value. Income, which we use as one of the three attributes, is not a functioning in Sen’s sense. Instead, it can be viewed as a proxy for many different functionings taken together, such as shelter from the elements, being well-nourished, social standing, etc. In this paper, however, we shall ignore such fine conceptual distinction between functionings and proxies for functionings. We shall use the terms "functionings", "attributes", and "dimensions of well-being" interchangeably.

\(^4\) The assumption that health, as well as education, takes a finite number of discrete values seems to be highly plausible. In principle, income can take an infinite number of values. Our data regarding income, however, comes in terms of 41 categories, represented by real intervals, such that each individual’s income belongs to exactly one of these categories (see Section 3 below).
elements in \( V_j \), and then taking a suitable positive linear transformation of these numbers so that the score for the lowest (resp. highest) ranking element of \( V_j \) becomes 0 (resp. 1). Note that under the usual definition, \( \#\{v'_j \in V_j: v_j \succ_j v'_j \} + 1 \) is the Borda score of an element \( v_j \) of \( V_j \) given the linear ordering \([\geq_j]\) over \( V_j \). For every \( j \in M \), the \([\geq_j]\)-least element of \( V_j \) will be denoted by \( v^0_j \) and the \([\geq_j]\)-greatest element of \( V_j \) will be denoted by \( v^*_j \). Let \( V = V_1 \times \ldots \times V_m \), and let \( v^0 = (v^0_1, \ldots, v^0_m) \) and \( v^* = (v^*_1, \ldots, v^*_m) \). Thus, \( v^0 \) is the individual achievement vector in which the achievement in every dimension is at the lowest level, and \( v^* \) is the individual achievement vector in which the achievement in every dimension is at the highest level.

### 2.2. The individual well-being function and inequality of well-being

Let \( N = \{1, 2, \ldots, n\} \) denote the society under consideration. A real-valued individual well-being function is a function \( \rho: V \rightarrow [0, \infty) \). Thus, for all \( i \in N \) and \( v \in V \), the well-being of \( i \) when \( i \)'s achieved functioning bundle is \( v \) is given by \( \rho(v) \). Note that, by definition, the individual well-being function \( \rho \) is invariant with respect to individuals.\(^5\) For each achievement matrix, \( A = (a_{ij})_{n \times m} \), \( \rho \) determines the vector, \((\rho(a_{1*}), \ldots, \rho(a_{n*}))\), of individual well-being levels and then we use a real-valued well-being inequality measure, \( H \), which specifies the degree of well-being

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\(^5\) The property that the individual well-being function \( \rho \) is invariant with respect to individuals can be interpreted either in terms of Sen’s (1987) ‘standard-evaluation’ (as opposed to what Sen (1987) calls ‘self-evaluation’) or in terms of the values advocated by the social scientist in the course of social deliberation. See, however, Decancq et al. (2017) for an approach to measuring multidimensional inequality in well-being where they assume that each individual’s well-being depends on her achievement vector as well as an individual-specific parameter reflecting her preferences.
inequality corresponding to \((\rho(a_1), ..., \rho(a_n))\). Thus, for every achievement matrix, \(A\), the inequality of well-being is given by \(Z(A) = H(\rho(a_1), ..., \rho(a_n))\).

There are several functional forms used to measure multidimensional well-being in the literature.\(^6\) Many have used the class of functions showing Constant Elasticity of Substitution (CES) to aggregate attributes in a multidimensional index (see Maasoumi (1986), Decancq and Lugo (2012), Rhode and Guest (2013) among others).\(^7\) Pattanaik and Xu (2019) provide an axiomatic characterization of a class of individual well-being functions \(\rho\) satisfying:

\begin{itemize}
  \item[(1)] there exist positive constants \(w_j (j \in M)\) with \(\sum_{j \in M} w_j = 1\), and there exists, for each \(j \in M\), an increasing function \(\phi_j: V_j \rightarrow [0, w_j]\), with \(\phi_j(v_j^0) = 0\) and \(\phi_j(v_j^*) = w_j\), such that, for some increasing function \(\sigma: [0,1] \rightarrow [0,1]\), with \(\sigma(0) = 0\) and \(\sigma(1) = 1\),
  \[\rho(v) = \sigma(\sum_{j=1}^{m} \phi_j(v_j))\] for all \(v \in V\).\(^8\)
\end{itemize}

In this paper we use a particular subclass of the class of individual well-being functions satisfying (1). This is the class of \(\rho\) functions which satisfy (2) below:

\begin{itemize}
  \item[(2)] there exist positive constants \(w_j (j \in M)\) with \(\sum_{j \in M} w_j = 1\), such that, for every \(v \in V\),
  \[\rho(v) = [\sum_{j \in M} w_j s(v_j)]^\gamma\] for some positive \(\gamma\).\(^9\)
\end{itemize}

\(^6\) In addition to functional forms, multidimensional indices have been measured using multivariate techniques, such as principal components (Maasoumi and Nickelsburg, 1988) and cluster analysis (Hirschberg et al., 1991).\(^7\) The Cobb-Douglas function and the linear utility function are special cases of the CES function. Bourguignon and Chakravarty (2003) argued that the CES form of aggregation implies the same elasticity of substitution between all dimensions. In the Cobb-Douglas form, the elasticity of substitution is equal to one, whereas in the linear form, attributes are perfect substitutes of each other.\(^8\) Pattanaik and Xu’s (2019) axiomatic characterization of individual well-being functions \(\rho\) satisfying (1) emerges as a byproduct of their characterization of a class of social well-being functions in terms of properties of social well-being functions. It is, however, easy to see that one can readily adapt their axioms to provide a direct axiomatization of individual well-being functions \(\rho\) satisfying (1) in terms of properties imposed on \(\rho\) itself. Pattanaik and Xu (2021) provide such a direct axiomatization of \(\rho\) functions satisfying (1).\(^9\) It may be of interest to note that, in our context, a CES individual well-being function may be written as \([\sum_{j \in M} w_j (s(v_j))^\lambda]\). Clearly, in general, an individual well-being function, which satisfies (2) with \(\gamma \neq 1\), is different from a CES type individual well-being function.

\(^8\) Pattanaik and Xu (2021) provide such a direct axiomatization of \(\rho\) functions satisfying (1).
Bossert et al. (2013), Dhongde et al. (2016) and Dhongde et al. (2019) are some earlier contributions which have also used individual well-being functions $\rho$ satisfying (2). $w_j (j \in M)$ figuring in (2) can be interpreted as reflecting the relative importance attached to attribute $f_j$. One can, then, interpret $\sum_{j \in M} w_j s(v_j)$ as an individual’s index of overall achievement (IOA) in terms of the different attributes when her achievement vector is given by $v \in V$. Given an individual well-being function satisfying (2), where the individual’s well-being is an increasing function of her IOA, should we just consider the distribution of individual IOA rather than the distribution of individual well-being? Obviously, if $\gamma = 1$, it will not matter whether we measure inequality in the distribution of IOA or the distribution of individual well-being. But, if $\gamma \neq 1$, the assessment of inequality in the society will, in general, differ, depending on whether we consider inequality of individual IOAs or inequality of individual well-being. When this happens, we believe that, it is inequality in the distribution of individual well-being, which should be taken into account in the context of social welfare judgments. Once we derive the well-being levels, $\rho(a_1), \rho(a_2), \ldots, \rho(a_n)$, of the different individuals for a given achievement matrix $A = (a_{ij})_{n \times m}$, we can use a widely accepted inequality measure, such as the Gini index to measure inequality in the distribution of individual well-being.

Two points may be noted about (2). First, for every $j \in M$, the ‘adjusted’ Borda scores, $s(v_j)$ ($v_j \in V_j$), can be viewed as a cardinal real-valued representation of the linear ordering $[\geq_j]$ over $V_j$ such that the numbers assigned to $v_j^0$ and $v_j^*$ are 0 and 1 respectively. It is easy to check that one can have other real-valued functions representing the linear ordering $[\geq_j]$ over $V_j$, which assign the numbers 0 and 1 to $v_j^0$ and $v_j^*$, respectively. We have chosen to use the
adjusted Borda scores because of their simplicity\(^\text{10}\), but we do recognize the need for trying out alternative cardinal real-valued representations of \([\geq j]\) over \(V_j\ (j \in M)\) in empirical work so as to gain insight into the implications of these different representations in our context.

Second, the parameter \(\gamma\ (\gamma > 0)\) in (2) will be assumed to be less than 1 in this paper. The magnitude of \(\gamma\) has often been linked to the notions of substitutability, complementarity or independence of two attributes. Given an individual well-being function \(\rho\), which satisfies two attributes, \(f_k\) and \(f_{k'}\) are said to be substitutes (resp. complements) if the second-order cross-partial derivative is negative (resp. positive) for those two attributes; and the two attributes are said to be independent when the second order cross partial derivative of the individual well-being function \(\rho\) is zero for the two attributes. Given these notions of substitutability, complementarity and independence of attributes, it is clear that, for an individual well-being function \(\rho\) satisfying (2), \(\gamma < 1\) (resp. \(\gamma > 1\)) if and only if the attributes are all substitutes (resp. complements) of each other in the sense mentioned above (see Pattanaik et al. (2012) and Dhongde et al. (2019)). The notion of substitutes as defined above is of interest and provides an interpretation for our assumption that \(\gamma < 1\). But the fact that \(\gamma < 1\) if and only if all attributes are substitutes of each other does not provide a strong enough direct intuitive justification for assuming \(\gamma < 1\). A more transparent intuitive justification for assuming \(\gamma < 1\), however, can be given since it can be shown that for an individual well-being function, \(\rho\), satisfying (2), \(\gamma < 1\) if and only if \(\rho\) satisfies (3) below\(^\text{11}\):

\(^{10}\) However, note that some contributions (see, for instance, van Doorslaer and Jones, 2003) in the empirical literature on health have suggested that the assumption of "equal-distance" between successive levels of a self-assessed health scale is problematic; this assumption is clearly built into our use of the adjusted Borda scores for health.

\(^{11}\) A detailed discussion of this and other related results can be found in Pattanaik and Xu (2021).
(3) starting from a given initial situation, the fall in an individual’s well-being that results when, other things remaining the same, the individual’s achievements in terms of any two attributes, \( f_k \) and \( f_{k'} \), fall simultaneously by \( \varepsilon_k \) and \( \varepsilon_{k'} \) (\( \varepsilon_k > 0, \varepsilon_{k'} > 0 \)), respectively, is greater than the sum of: (i) the fall in the individual’s well-being when, starting from the same initial situation, the individual’s achievement in terms of \( f_k \) declines by \( \varepsilon_k \), the individual’s achievements in terms of all other attributes remaining the same, and (ii) the fall in the individual’s well-being when, starting again from the same initial situation, the individual’s achievement in terms of \( f_{k'} \) declines by \( \varepsilon_{k'} \), the individual’s achievements in terms of all other attributes remaining the same.

We believe that, for an individual well-being function \( \rho \) satisfying (2), requirement (3), which is analogous to a property proposed by Stiglitz, Sen and Fitoussi (2009) for measures of deprivation, is highly persuasive. Since an individual well-being function \( \rho \) satisfying (2) satisfies (3) if and only if \( \gamma < 1 \), (3) provides a direct and transparent intuitive justification for the assumption that \( \gamma < 1 \).

2.3. The Type I approach to measuring multidimensional inequality and its limitations

In our empirical analysis in the next section, we use the procedure outlined in Section 2.2. It obviously comes within what we have called the Type II approach to the measurement of well-being inequality. It may be useful to discuss a little further why we have chosen not to adopt the Type I approach to measuring inequality of well-being though we do believe that it may give us otherwise useful information. The intuitive difficulty involved in the Type I approach to measuring inequality has been noted earlier by Decancq et al. (2017). Analogous problems in
the context of the measurement of deprivation and the standard of living have been discussed by Dutta et al. (2003).

Consider the Type I approach. In this approach, we first choose, for every $j \in M$, a real-valued inequality measure $\vartheta_j$, which, for every vector, $c \cdot j$, of individual achievements along dimension $f_j$ specifies a real number $\vartheta_j(c \cdot j)$ as the index of inequality in the individuals’ achievements in terms of $f_j$. Next, given an achievement matrix, $C$, one aggregates the dimensional inequalities, $\vartheta_1(c \cdot 1), \vartheta_2(c \cdot 2), \ldots, \vartheta_m(c \cdot m)$ so as to reach a real number that serves as the index of inequality reflected in the entire achievement matrix, $C$. Let $K$ be the set of all $m$-tuple of real numbers $(y_1, \ldots, y_m)$ such that, for all $j \in M$, $y_j = \vartheta_j(c \cdot j)$ for some achievement matrix $C$. Consider the class of all real-valued functions defined on $K$. One needs to choose a function $G$ from this class to aggregate the dimensional inequality indices. Once such a function $G$ is chosen, the overall inequality for any achievement matrix $C$ is given by $Y(C) = G(\vartheta_1(c \cdot 1), \vartheta_2(c \cdot 2), \ldots, \vartheta_m(c \cdot m))$.

When we are engaged in ethical evaluation of social states, inequality of well-being distribution is an important consideration and, to assess inequality of well-being, it makes sense to find first the well-being of each individual and then to assess inequality in the distribution of individual well-being, i.e., to use a Type II inequality measure $Z(.)$ defined in terms of a suitably chosen individual well-being function $\rho(.)$ and a suitably chosen measure, $H$, of well-being inequality, as described in Section 2.2. But an interesting question arises here. Suppose we have a “satisfactory” Type II inequality measure $Z(.)$. Can we possibly find a Type I inequality measure $Y(.)$ such that, for all achievement matrices, $A$ and $B$, $[Y(A) \geq Y(B)]$ if and only if $Z(A) \geq Z(B)$?
If this can be done, then, for most practical purposes, it would not matter whether we use the Type II measure $Z(.)$ of inequality of individual well-being, or we use a Type I measure $Y(.)$ of inequality that will give us the same ranking of different possible achievement matrices as the ranking that we get by using $Z(.)$. This, however, cannot be done, given some very mild assumptions. Thus, under some very mild assumptions, every Type I measure of inequality will deviate from the ranking of some achievement matrices given by the Type II measure of well-being, namely $Z(.)$. A detailed and general study of this problem needs separate investigation and is beyond the scope of this paper. But a simple example will illustrate the basic point.

**Example 1.** Let $N = 2$ and let $M = 2$. Suppose, for each $j \in M$, there are only two possible values, 1 (satisfactory) and 0 (unsatisfactory), that an individual’s achievement in terms of attribute $f_j$ can take. Suppose we have a Type II measure of well-being inequality $Z(.)$ such that, for every achievement matrix $C$, $Z(C) = H(\rho(c_{11}, c_{12}), \rho(c_{21}, c_{22}))$, where:

(4) $\rho$ is increasing in the individual’s achievement in each dimension;

(5) $H$ is such that, for all achievement matrices $A = (a_{ij})_{2 \times 2}$, and $B = (b_{ij})_{2 \times 2}$, if $\rho(a_{1*}) > \rho(b_{1*}) > \rho(a_{2*})$ and $\rho(a_{1*}) > \rho(b_{2*}) > \rho(a_{2*})$, then $Z(A) > Z(B)$.

(4) just embodies the intuition that each attribute is a positively valued attribute. (5) is an intuitively compelling property for a measure of well-being inequality in our two-person society.

Now consider a Type I measure $Y(.)$ such that, for every achievement matrix $C$, $Y(C) = G(\vartheta_1(c_{1*}), \vartheta_2(c_{2*}))$, where the functions $G$ and $\vartheta_j$ ($j = 1, 2$) have the interpretations as given earlier in this section. The only restriction that we postulate here is the following property of
“anonymity” (capturing the idea that, for each dimension, two achievement vectors having the same distribution yield the same inequality index) for the functions \( \vartheta_1 \) and \( \vartheta_2 \):

(6) for every \( j \in \{1, 2\} \) and any two vectors, \( c_j \) and \( d_j \), of individual achievements along dimension \( f_j \), if \( d_{1j} = c_{2j} \) and \( d_{2j} = c_{1j} \), then \( \vartheta_j(c_j) = \vartheta_j(d_j) \).

To see that the inequality ranking of achievement matrices implied by \( Y(.) \) does not coincide with that implied by \( Z(.) \), consider the following two achievement matrices, \( A \) and \( B \). The rows show the individuals and the columns show the attributes. Matrix \( A \) is given by \[
\begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix}
\]
and Matrix \( B \) is given by \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]. Note that \( a_{*1} = b_{*1} \) so that \( \vartheta_1(a_{*1}) = \vartheta_1(b_{*1}) \). By (6), \( \vartheta_2(a_{*2}) = \vartheta_2(b_{*2}) \) (note that \( a_{*2} \) and \( b_{*2} \) have the same distribution). Since \( G \) is a function, it follows that \( Y(A) = G(\vartheta_1(a_{*1}), \vartheta_2(a_{*2})) = G(\vartheta_1(b_{*1}), \vartheta_2(b_{*2})) = Y(B) \). By (4), \( \rho(a_{*1}) > \rho(b_{*1}) > \rho(a_{*2}) \) and \( \rho(a_{*1}) > \rho(b_{*2}) > \rho(a_{*2}) \). Hence, by (5), we have \( Z(A) > Z(B) \). Thus, we have \( Y(A) = Y(B) \), but \( Z(A) > Z(B) \).

3. DATA

We use data from the Current Population Survey (CPS) conducted jointly by the United States Census Bureau and the Bureau of Labor Statistics for over 50 years. The Annual Social and Economic (ASEC) Supplement of the CPS is used to estimate the official poverty measure (OPM) as well as the supplemental poverty measure (SPM). The CPS is a nationally representative household survey that collects individual and household level data on income sources, and other indicators such as work experience, poverty, health insurance coverage, and education.
We use data from individual records for three years, 2010, 2014 and 2019.\textsuperscript{12} We choose 2011 March CPS-ASEC round since that is the first year when SPM adjusted income is available with NBER; 2020 March CPS-ASEC round is the latest round available and is the last year before the Covid-19 pandemic hit the economy.

### 3.1. Dimensions of Well-being

In Table 1, we summarize a sample of empirical studies, which have previously used the two-step approach to measure well-being inequality using individual level data.\textsuperscript{13} Most of the studies use panel data on households or individuals (except Glassman, 2019) and all of the listed studies follow Massoumi (1986) and use the CES function to aggregate an individual’s multiple dimensions.

**Table 1: Sample of Empirical Studies using the Two-Step Approach with Individual Data**

<table>
<thead>
<tr>
<th>Study</th>
<th>Attributes</th>
<th>Multidimensional Individual Well-being Function</th>
<th>Inequality in Well-being</th>
<th>Country</th>
<th>Data-Years</th>
</tr>
</thead>
</table>
| Aristei and Bracalente, 2011 | 1. Income  
2. Predicted Health status  
3. Years of schooling | CES function | Generalized Atkinson Index | Italy | EU-SILC  
2005-2008 |
| Justino, 2012            | 1. Consump. expenditure  
2. Healthy days  
3. Years of schooling | CES function | Generalized Entropy Indices | Vietnam | VLSS  
1992-1997 |

\textsuperscript{12} Data for 2010, 2014 and 2019 are compiled from ASEC March 2011, 2015 and 2020 round respectively. https://www.census.gov/data/datasets/time-series/demo/cps/cps-asec.html

\textsuperscript{13} For applications of the two-step approach to multidimensional inequality using country wide data, see Decancq et al. (2009) and Jorda (2013).
2. Self-reported health status  
3. Years of education  

Rohde and Guest, 2018 | 1. Income | CES function | Theil Indices | United States, Germany, Australia | PSID, SOEP, HILDA 2001-2007  
2. Self-reported health status  
3. Years of education  
4. Leisure  

2. Self-reported health status  
3. Years of education  
4. Leisure  
5. Vehicle ownership  
6. Rooms per person  


The Commission on the Measurement of Economic Performance and Social Progress (Stiglitz et. al. 2009) recommended eight key attributes, including standard of living, education and health that should be taken into account simultaneously to define multi-dimensional well-being particularly in high-income countries. As seen in Table 1, most of the studies used the three dimensions that were also used in the Human Development Index, namely standard of living (income or consumption expenditure), education and health. Whenever data was available, a
few have included leisure and housing. We follow the literature and choose basic well-being attributes, namely, income, education and health, which are available in the CPS data.\textsuperscript{14} We use data on income adjusted for the Supplemental Poverty Measure (SPM). The SPM is an alternative poverty measure estimated since 2011 and overcomes many of the drawbacks of the OPM (see Fox 2020, for the latest SPM estimates). We use the SPM adjusted income since these take into account geographic differences in the cost of living and use equivalence scales to adjust for households with different sizes and compositions (Bridges and Gesumaria, 2015, Garner and Gudrais, 2018). The SPM income measure is cash income plus in-kind government benefits (such as food stamps and housing subsidies) minus nondiscretionary expenses such as taxes and medical out-of-pocket expenses.

The CPS provides income categories for individual’s SPM income. We thus have 41 income categories; the lowest category has individuals with annual incomes less than $5,000. The income categories proceed in increments of $5,000 and the highest category is income greater than or equal to $200,000. Table 2 shows some of the income categories and the proportion of individuals belonging to these categories. The Great recession lowered overall income levels in 2010. Median incomes increased from $45,000-$49,999 in 2010 to $60,000-$64,999 in 2019. Over the decade, the proportion of individuals in lower income categories decreased and that in the top income categories increased.

\textsuperscript{14} These attributes are also included in most multidimensional poverty measures, in the United States (e.g. Dhongde and Haveman, 2017) and other high income countries such as Germany (e.g. Nowak and Scheicher, 2017), Australia (e.g. Martinez Jr. and Peralez, 2017) and in the European Union (e.g. Weziak-Bialowolska, 2016).
<table>
<thead>
<tr>
<th>No. of Categories</th>
<th>Attributes</th>
<th>Percent Pop. 2010</th>
<th>Percent Pop. 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Less than $5,000</td>
<td>3.0</td>
<td>2.4</td>
</tr>
<tr>
<td>2.</td>
<td>$5000 to $9,999</td>
<td>2.9</td>
<td>1.7</td>
</tr>
<tr>
<td>3.</td>
<td>$10,000 to $14,499</td>
<td>5.0</td>
<td>3.3</td>
</tr>
<tr>
<td>....</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>$50,000 to $54,499</td>
<td>5.2</td>
<td>4.5</td>
</tr>
<tr>
<td>12.</td>
<td>$55,000 to $59,999</td>
<td>4.4</td>
<td>4.2</td>
</tr>
<tr>
<td>....</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td>$100,000 to $104,999</td>
<td>1.6</td>
<td>2.1</td>
</tr>
<tr>
<td>22.</td>
<td>$105,000 to $109,999</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>....</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>$150,000 to $154,999</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>32.</td>
<td>$155,000 to $159,999</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>....</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39.</td>
<td>$190,000 to $194,999</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>40.</td>
<td>$195,000 to $199,999</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>41.</td>
<td>$200,000 and over</td>
<td>1.5</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Educational Attainment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Less than high school</td>
<td>12.2</td>
<td>8.8</td>
</tr>
<tr>
<td>2.</td>
<td>High school diploma or equivalent</td>
<td>28.4</td>
<td>25.7</td>
</tr>
<tr>
<td>3.</td>
<td>Some college but no degree</td>
<td>19.4</td>
<td>17.0</td>
</tr>
<tr>
<td>4.</td>
<td>Associate degree</td>
<td>9.8</td>
<td>10.6</td>
</tr>
<tr>
<td>5.</td>
<td>Bachelor’s degree</td>
<td>19.1</td>
<td>23.0</td>
</tr>
<tr>
<td>6.</td>
<td>Masters, Professional, Doctorate degree</td>
<td>11.2</td>
<td>14.9</td>
</tr>
<tr>
<td><strong>Health Status</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Poor</td>
<td>3.5</td>
<td>2.6</td>
</tr>
<tr>
<td>2.</td>
<td>Fair</td>
<td>8.3</td>
<td>7.8</td>
</tr>
<tr>
<td>3.</td>
<td>Good</td>
<td>24.6</td>
<td>24.3</td>
</tr>
<tr>
<td>4.</td>
<td>Very Good</td>
<td>31.4</td>
<td>32.8</td>
</tr>
<tr>
<td>5.</td>
<td>Excellent</td>
<td>32.2</td>
<td>32.6</td>
</tr>
</tbody>
</table>

Notes: Distribution of attributes in 2014 is not included for brevity. We include all individuals, including the elderly and the children; children under the age of 18 are assigned maximum educational attainment within the household; SPM adjusts income using equivalence scales for children.
The CPS has data on individual’s educational attainment. Between 2010 and 2019, the proportion of individuals with lower attainment levels (less than a degree) decreased, and those with higher attainment increased. For example, compared to the 11.2% in 2010, 14.9% individuals had a masters, professional or doctorate degree in 2019. In fact, there is a first-order stochastic dominance observed; the cumulative distribution in 2019 dominates the 2010 distribution.

There are five categories for self-assessed health given in the CPS. The survey collects data on individual’s overall health status by asking them to rank their health on a scale of 1 to 5. Unlike income and education, in both years, a majority (about 64%) chose health status as very good or excellent.

3.2. Set-up for a Benchmark Well-being Index

Given that income has 41 categories, education has six categories and health has five categories, we transform the categorical data for each attribute into scores ranging from 0 to 1. Thus, income categories are transformed as \( \left\{ 0, \frac{1}{40}, \frac{2}{40}, ..., 1 \right\} \), educational attainment levels are transformed as \( \left\{ 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1 \right\} \) and the five categories of health status are transformed as \( \left\{ 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1 \right\} \). The IOA of an individual is given by \( \sum_{j \in M} w_j s(v_j) \) and her well-being function is given by \( \rho(v) = \left[ \sum_{j \in M} w_j s(v_j) \right]^\gamma \). In the benchmark case, we assign equal weights to all three attributes, so that \( w_j = \frac{1}{3}, j = 1, 2, 3 \) and assume \( \gamma = \frac{1}{2} \) in the well-being function so that, as the individual’s IOA increases, her well-being increases but at a diminishing rate. We vary weights assigned to each of the attributes to test the sensitivity of the results.
4. INEQUALITY IN MULTIDIMENSIONAL WELL-BEING IN THE U.S.

4.1. Changes in Inequality over time

We estimate three inequality indices for each attribute (normalized between 0 and 1) and for multidimensional well-being. The Gini index is the most commonly used measure of inequality. However, it is sensitive to differences in attributes near the mode of the distribution. Hence, we also estimate inequality measures, which tend to be sensitive to tails of the distribution.\(^\text{15}\) In addition to the Gini index, we also estimate the Generalized Entropy index (GE1), also known as the Theil index and the Atkinson (A-0.5) index.

In Table 3, we show the percentage changes in these indices over time. The different inequality indices show similar trends. Income inequality rose during the Great Recession between 2010 and 2014, but then declined between 2014 and 2019. On the other hand, inequality in educational attainment continually declined throughout the decade. There was not much change in the distribution of health status in first half of the decade but the distribution became more even in the second half.

Given the different directional changes in inequality in each of the three attributes, we are interested to find out how inequality in the multidimensional well-being index based on these attributes changed. As seen in Table 3, inequality in well-being increased between 2010 and 2014 and decreased between 2014 and 2019. The joint distribution of the three attributes became more unequal during the Great Recession and less unequal during the recovery period.

\(^{15}\) In a generalized entropy index GE(t), when t>0, the index is sensitive to the top of the distribution, and when t<0, the index is sensitive to the bottom of the distribution. In the Atkinson’s index A(e), the more positive e > 0 is, the more sensitive A(e) is to differences at the bottom of the distribution.
Table 3. Change in Inequality in Attributes and in Multidimensional Well-being

<table>
<thead>
<tr>
<th>Inequality Index</th>
<th>% Change</th>
<th>Income</th>
<th>Education</th>
<th>Health</th>
<th>Multidimensional Well-being</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010-2014</td>
<td>2.4</td>
<td>-1.2</td>
<td>0.1</td>
<td></td>
<td>9.1</td>
</tr>
<tr>
<td>2014-2019</td>
<td>-1.7</td>
<td>-4.8</td>
<td>-2.7</td>
<td></td>
<td>-2.8</td>
</tr>
<tr>
<td><strong>2010-2019</strong></td>
<td><strong>0.6</strong></td>
<td><strong>-5.9</strong></td>
<td><strong>-2.6</strong></td>
<td></td>
<td><strong>6.1</strong></td>
</tr>
<tr>
<td>Theil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010-2014</td>
<td>5.2</td>
<td>-2.0</td>
<td>0.0</td>
<td></td>
<td>19.2</td>
</tr>
<tr>
<td>2014-2019</td>
<td>-5.6</td>
<td>-8.0</td>
<td>-5.8</td>
<td></td>
<td>-7.1</td>
</tr>
<tr>
<td><strong>2010-2019</strong></td>
<td><strong>-0.7</strong></td>
<td><strong>-9.8</strong></td>
<td><strong>-5.7</strong></td>
<td></td>
<td><strong>10.8</strong></td>
</tr>
<tr>
<td>Atkinson</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010-2014</td>
<td>4.6</td>
<td>-1.7</td>
<td>0.0</td>
<td></td>
<td>20.1</td>
</tr>
<tr>
<td><strong>2010-2019</strong></td>
<td><strong>1.5</strong></td>
<td><strong>-8.7</strong></td>
<td><strong>-6.1</strong></td>
<td></td>
<td><strong>10.6</strong></td>
</tr>
</tbody>
</table>

Note: We provide estimates of inequality indices in the Appendix Table A1.

Next, we illustrate that the difference between Type I and Type II approaches can be empirically relevant (also see Decancq and Lugo (2012)'s illustration with Russian data). Figure 1 shows the percentage change in the Gini index for each of these attributes between 2010 and 2019. In the Type I approach, we take a simple average of the Gini indices of the three attributes. We find that the average Gini declined by about 2.2 percent. However, if we use the three attributes first to estimate an index of multidimensional well-being for each individual and then measure the percentage change in the Gini index, we find that inequality in well-being in fact increased by 6.1 percent. Thus, the Type I and Type II approaches can provide very different results when measuring inequality in a multidimensional framework.
4.2. Inequality in Multidimensional Well-being Using Alternate Weights on Attributes

Inequality in multidimensional well-being in Table 3 is estimated by assigning equal weights to all three attributes. We now consider what happens when we change the weights on these attributes.\(^\text{16}\)

In Table 4, we use three alternate weighting schemes and calculate changes in the Gini index over time. In the first panel, we assign zero weight to one attribute and equal (1/2) weights to the remaining two attributes. For example, when we assign zero weight to income, we see that inequality in multidimensional well-being in fact decreased by 6.1% from 2010 to 2019. A rise in income inequality was a contributing factor in increasing inequality in well-being. On the other hand, in the third panel, when we increase the weight on income to 1/2 and assign 1/4 weight

\(^{16}\) We also vary values of \(\gamma (\gamma = 1,2)\) and find that the trends in well-being inequality are robust to these changes. Recall that when \(\gamma = 2\), attributes are complements and when \(\gamma = 1\), attributes are independent (the second-order cross-partial derivatives are zero).
to the other two attributes, inequality in multidimensional well-being increased significantly by 12.6%. When equal weights are placed on all attributes in the second panel and the values show are similar to those shown in Table 3. Recall from Table 3 that inequality in education declined over time. Therefore, when we place a greater weight on education, between 2010 and 2019, well-being inequality increased by the least percentage (1.4%) and when we remove education from the well-being index, well-being inequality increased significantly (9.5%).

Table 4. Gini Indices of Multidimensional Well-being using Alternate Weights on Attributes

<table>
<thead>
<tr>
<th>% Change</th>
<th>(%I=0, 1/2, 1/2)</th>
<th>(%1/2, E=0, 1/2)</th>
<th>(%1/2, 1/2, H=0)</th>
<th>Benchmark (%1/3, 1/3, 1/3)</th>
<th>(%I=1/2, 1/4, 1/4)</th>
<th>(%1/4, E=1/2, 1/4)</th>
<th>(%1/4, 1/4, H=1/2,)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010-2014</td>
<td>-1.2</td>
<td>9.0</td>
<td>10.6</td>
<td>9.1</td>
<td>12.9</td>
<td>6.1</td>
<td>6.2</td>
</tr>
<tr>
<td>2014-2019</td>
<td>-5.0</td>
<td>0.5</td>
<td>-7.3</td>
<td>-2.8</td>
<td>-0.3</td>
<td>-4.5</td>
<td>-3.4</td>
</tr>
<tr>
<td>2010-2019</td>
<td>-6.1</td>
<td>9.5</td>
<td>2.6</td>
<td><strong>6.1</strong></td>
<td><strong>12.6</strong></td>
<td><strong>1.4</strong></td>
<td><strong>2.6</strong></td>
</tr>
</tbody>
</table>

Notes: Weights attached show those for I: Income, E: Educational Attainment and H: Health status.

4.3. Distribution of Attributes Conditional on Income

We analyze the distribution of each of the attributes conditional on income, in order to understand how an increase in inequality in multidimensional well-being may have come about.

In Table 5, we provide median values of each of the three attributes (since these are categorical variables) by income deciles.

During the Great Recession (2010-2014), median income did not increase for a majority of deciles except the lowest and the highest decile. Median income grew in the second half of the decade from 2014 to 2019, except in the lowest decile. Over the entire decade, median income for the lowest decile grew only marginally from income category 2 to 3, i.e. from $5000-$9999
to $10,000-$14,999. On the other hand, median incomes in the top decile increased from 
income category, 28 to 41, i.e. from $135,000-$139,999 to over $200,000.

There was not much variance in the median values of health status; most income deciles had 
their health status as very good health. However, the distribution on education by income 
deciles changed over time. In 2010, median education in each income decile was 3, that is, 
some college but no degree. In 2019, however, median education increased among higher 
income deciles. In income deciles 6 and 7 median education increased to associate degree and 
in deciles 8, 9 and 10, it increased to bachelor’s degree. Thus, conditional on incomes, the 
distribution of educational attainment became more unequal, although the unconditional 
distribution of educational attainment became more equal over time as seen in Table 3.

Table 5. Distribution of Attributes by Income Deciles

<table>
<thead>
<tr>
<th>Decile</th>
<th>2010</th>
<th>2014</th>
<th>2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>28</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: Table shows median values of categories of income, education and health.

This overlap of income and educational attainment and the associated rise in inequality is 
captured by the change in the distribution of multidimensional well-being index shown in Table 
6. During the Great Recession, between 2010 and 2014, well-being decreased for the lowest
three income deciles, it remained unchanged for the middle deciles and increased only for the top four deciles. During the recovery period, between 2014 and 2019, multidimensional well-being increased for most deciles except the lowest income decile.

Over the decade (2010-2019), multidimensional well-being increased significantly for the top three income deciles; the rise was by more than 12 percent. For the middle income deciles, namely, decile 6 and 7, well-being increased by 7.5 percent and 9.5 percent respectively. However, if we look at the lower income deciles, namely deciles 3, 4 and 5, well-being increased by less than 5%. Well-being in fact decreased for the lowest two deciles. It seems that the major contributors for the significant increases in well-being for the top three income deciles are income and education, and the major contributor for the decrease in well-being for the lowest two deciles is education (their incomes increased slightly).

Thus, we realize that the distribution of individuals’ well-being index captures inequality in the joint distribution of attributes, which is missed when we measure inequality in the marginal distribution of each attribute separately.

Table 6. Percentage Change in Multidimensional Well-being by Income Deciles

<table>
<thead>
<tr>
<th>Decile</th>
<th>2010 to 2014</th>
<th>2014 to 2019</th>
<th>2010 to 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.9</td>
<td>-1.3</td>
<td>-9.1</td>
</tr>
<tr>
<td>2</td>
<td>-7.4</td>
<td>2.4</td>
<td>-5.2</td>
</tr>
<tr>
<td>3</td>
<td>-5.9</td>
<td>7.3</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>6.6</td>
<td>7.5</td>
</tr>
<tr>
<td>7</td>
<td>4.1</td>
<td>5.2</td>
<td>9.5</td>
</tr>
<tr>
<td>8</td>
<td>5.4</td>
<td>6.2</td>
<td>12.0</td>
</tr>
<tr>
<td>9</td>
<td>4.9</td>
<td>7.0</td>
<td>12.3</td>
</tr>
<tr>
<td>10</td>
<td>9.5</td>
<td>3.1</td>
<td>12.9</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

In the last decade, multidimensional poverty indices have been widely used to measure deprivation in the quality of life often not captured by income poverty measures. In this paper, we assessed inequality in multidimensional well-being. We distinguished between two alternative approaches to measure such inequality. The Type I approach calculates an average of inequality indices for the separate attributes. The Type II approach, on the other hand, calculates an inequality index based on the multidimensional well-being of individuals.

Using data on three basic dimensions of well-being, namely, income, education and health, we studied changes in the inequality of multidimensional well-being of individuals in the United States in the last decade. The Type I approach showed that multidimensional inequality had slightly decreased but the Type II approach showed that, in fact, inequality in multidimensional well-being had increased. This is consistently observed in different inequality measures. Our analysis revealed that over time, the joint distribution of income and educational attainment had become more unequal. This was reflected in the Type II approach to measure inequality since the multidimensional well-being index captured the overlap of attributes experienced by an individual; the Type I approach failed to measure this overlap. The paper, thus, highlights the importance of measuring multidimensional well-being and its distribution in order to get a comprehensive picture of changes in the quality of life in a society over time.
References


Appendix

Table A1: Indices measuring Inequality in Attributes and in Multidimensional Well-being

<table>
<thead>
<tr>
<th>Inequality Index</th>
<th>Years</th>
<th>Income</th>
<th>Education</th>
<th>Health</th>
<th>Multidimensional Well-being</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td>2010</td>
<td>36.419</td>
<td>27.294</td>
<td>14.890</td>
<td>11.113</td>
</tr>
<tr>
<td></td>
<td>2014</td>
<td>37.277</td>
<td>26.973</td>
<td>14.899</td>
<td>12.121</td>
</tr>
<tr>
<td></td>
<td>2019</td>
<td>36.653</td>
<td>25.683</td>
<td>14.504</td>
<td>11.786</td>
</tr>
<tr>
<td>Theil</td>
<td>2010</td>
<td>21.771</td>
<td>12.156</td>
<td>4.268</td>
<td>2.140</td>
</tr>
<tr>
<td></td>
<td>2014</td>
<td>22.91</td>
<td>11.91</td>
<td>4.270</td>
<td>2.551</td>
</tr>
<tr>
<td></td>
<td>2019</td>
<td>21.627</td>
<td>10.961</td>
<td>4.024</td>
<td>2.371</td>
</tr>
<tr>
<td>Atkinson</td>
<td>2010</td>
<td>10.935</td>
<td>6.289</td>
<td>2.302</td>
<td>1.137</td>
</tr>
<tr>
<td></td>
<td>2014</td>
<td>11.443</td>
<td>6.183</td>
<td>2.302</td>
<td>1.365</td>
</tr>
<tr>
<td></td>
<td>2019</td>
<td>11.104</td>
<td>5.743</td>
<td>2.161</td>
<td>1.257</td>
</tr>
</tbody>
</table>