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From economic productivity to productive well-being: the role of life satisfaction and adjusted net savings*

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Abstract

Productivity - a driver of economic growth – is not necessarily compatible with societal well-being, nor environmental sustainability. Various authors contributed frameworks to incorporate environmental issues in the computation of productivity, or studied the role of subjective well-being for productivity. However, studies proposing ways to account for both subjective well-being and sustainability in productivity measurement are scarce. We check whether and to what extent it is possible to include subjective well-being and sustainability measures among the inputs and/or outputs of a traditional productivity framework. Specifically, we adopt a data-driven approach to test whether subjective well-being and adjusted net savings meaningfully contribute to computing a productivity-like indicator. We apply Data Envelopment Analysis (DEA) to European data from 2005 to 2018. We find that including subjective well-being among the inputs and the outputs of production meaningfully contributes to the measurement of total factor productivity.
Keywords: DEA, life satisfaction, adjusted net savings, productivity, Europe.
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1 Introduction

Productivity, i.e. the ratio of goods and services produced (outputs) divided by resources used in the production process (inputs), is usually considered a core indicator of economic performance, and a proxy of improving living conditions when it increases. Productivity provides a measure of how efficiently a production process uses scarce resources and develops new technologies. Enhancing productivity means making better use of available resources, and mobilising new technological potential to provide more or better goods and services to the society. Hence, productivity is often regarded as the ultimate engine of growth, and a measure for technical progress. In fact, it is usually held that expanding the set of goods and services available for consumption allows people to satisfy a growing number of needs, thus improving their living conditions (Solow, 1956). However, the efficient mobilization of resources for economic output and technological change does not imply societal well-being, nor environmental sustainability. These aspects are important and, in case of sustainability, urgent for modern societies.

Numerous authors warned that growing productivity does not necessarily translate into improved living conditions or environmental quality. For instance, waste and pollution are two negative sides of production processes. Moreover, since the COP 21 meeting held in Paris in 2015 – where most of the countries commited to achieve sustainability goals – sustainability can be regarded as a desirable output of economic activity, and integrated in productivity indicators. Accordingly, many authors proposed frameworks for efficiency/productivity indicators to account, for instance, for pollution as an undesirable by-product of production (an early attempt in this regard is Pittman (1983)). Zhou et al. (2018) provide a survey of some frameworks used to introduce sustainability in productivity measurement. A recent example is DiMaria (2019), who included adjusted net savings (ANS), an indicator of weak sustainability and welfare[1], in the set of desirable outputs. Conversely, studies proposing ways to account for both subjective well-being and sustainability in productivity measurement are scarce.

We contribute to this literature by applying a data-driven approach to establish whether and to what extent it is possible to extend the inputs and outputs of a traditional productivity framework to include subjective well-being and sustainability measures. We expect subjective well-being to be an input because of its positive association with productivity documented in previous literature. Additionally, we check whether subjective well-being and adjusted net savings can be outputs. If the production process delivers goods and services to satisfy people's needs, then we should expect a positive contribution of production to subjective well-being. Similarly, if the production process is environmentally sustainable, then adjusted net savings should be one of its outcomes. We posit that it is important to evaluate how well economies deliver goods and services given the resources they use. At the same time, we seek to go "beyond GDP", and to include measures of subjective well-being and environmental quality among economic indices of performance of "inclusive growth". In the framework of productivity measurement this means classifying subjective well-being either as an input, an output or both; it also implies checking whether sustainability is a desirable by-product of economic production.

This research is relevant because, if confirmed, it would suggest the existence of a virtuous cycle where investing in life satisfaction, by prioritizing social relations and environmental quality, would contribute to economic productivity (Sarracino, 2019). However, the resulting economic growth would be *qualitatively* different from the traditional one, and arguably more socially and environmentally sustainable (Sarracino and O'Connor, 2021b).

[1] ANS is an indicator of sustainability that translates sustainability and welfare gains as explained in Hamilton and Clemens (1999).

The analysis builds on a procedure for optimal selection proposed by Toloo et al. (2021). The procedure uses linear programming to compute optimal weights for the aggregation of outputs and inputs, including subjective well-being and adjusted net savings. The test procedure allows us to tell whether a variable meaningfully contributes to a productivity indicator by checking the magnitude of weights: if a variable attracts a weight equal to zero, than it can not be considered as relevant for the productivity indicator. We find that life satisfaction should be regarded as an input for some countries, and as an output for others, whereas adjusted net savings do not appear to be a relevant output to benchmark countries. These results suggest that including life satisfaction among the inputs and the outputs of productivity could meaningfully contribute to the definition of a measure of economic performance that accounts for the quality of growth.

The paper is structured as follows. The next section summarizes the relevant literature and our contribution. Section 3 describes the method and data used in our analysis. Section 4 reports our findings: we first present the result of our optimal selection model; we then offer a classification of the considered countries based on classification tree; we finally use our results to compute a well-being adjusted Malmquist index of productivity. The last section summarizes our findings and discusses limits and advantages of the proposed measure of productivity.

2 Literature review

In recent years, the subjective well-being literature shed new light on the ability of economic growth to deliver better lives (Easterlin, 2017; Helliwell and Aknin, 2018; Sarracino and O'Connor, 2021a). Empirical evidence provided a nuanced view about the role of economic growth for subjective well-being, and suggested that quality of economic growth matters (Helliwell, 2016): if economic growth is compatible with a cohesive and inclusive society, it is reasonable to expect that well-being will improve (Easterlin, 2013; Oishi and Kesebir, 2015; Mikucka et al., 2017). In contrast, if economic growth leads to loneliness and inequality, well-being may arguably decline. This is consistent with the observation that the link between quality of life and affluence is, at best, weak (Lovell et al., 1994; Beja, 2014).

Subjective well-being is the result of the presence of positive emotions, the absence of negative ones and satisfaction with life as a whole (Diener et al., 1999). In practice, however, subjective well-being is frequently monitored through one of its components: life satisfaction, which is regarded as an evaluative and cognitive measure of subjective well-being. This individual level information is usually collected in the course of surveys, when respondents are asked questions such as: "All things considered, how satisfied are you with your life as a whole these days?" (Van Praag et al., 2003). Answers usually range on a scale where low/high scores indicate total dis/satisfaction. Various tests, from different disciplines, provided evidence supporting the validity and reliability of life satisfaction as a measure of how people fare with their lives (Blanchflower and Oswald, 2004; Van Reekum et al., 2007; Schimmack et al., 2010; Kahneman and Krueger, 2006; Layard, 2005).

The relationship between productivity and measures of well-being received particular attention in the economic literature. For instance, (Edmans, 2011) documents that companies in which employees' satisfaction is higher receive higher long-run stock returns. Studies on subjective well-being on the workplace report a positive association with various measures of productivity[2] using matched employer-employee panel data from Finland (Böckerman and Ilmakunnas, 2012), and Great Britain (Bryson et al., 2017). The results hold both in levels and first differences. Furthermore, Oswald et al. (2015) showed that happiness increases productivity in three different experimental settings. According to the authors, productivity gains are due to the fact that satisfied people are more committed to their tasks than others. However, few studies have tried to merge productivity and subjective well-being in one composite indicator of economic performance. For instance, DiMaria et al. (2020) evaluated whether life satisfaction (as an input or an output) contributed to efficiency following a procedure proposed by Pastor et al. (2002), and four waves of the European Social Survey (2004, 2006, 2008, and 2010). Results indicate that for some countries, mainly Western European economies, the stock of employees satisfied with their lives should be regarded as an input, and therefore it belongs to the denominator of productivity computations. For Eastern European countries the stock of satisfied people is more likely to be an output, and therefore it belongs to the numerator of productivity indexes.[3]

The starting point of our analysis is the usual definition of productivity

[2]Böckerman and Ilmakunnas (2012) consider the following measures of productivity: value added per hours worked, total factor productivity, and turnover per employee; Bryson et al. (2017) use financial performance, labour productivity, quality of product or service, and a performance scale summing up the three previous measures.

[3]An alternative specification of our model would be to use subjective well-being as a multiplier of labor, similarly to human capital. However, the results from the new specification would indicate whether labor or labor multiplied by subjective well-being should be regarded as input. In the present model we require labor to be always an input of productivity, and we check whether – in addition – the stock of employees satisfied with their lives contributes to the measure of productivity.

indicators as outputs divided by inputs, where outputs are GDP (to account for economic performance), life satisfaction and adjusted net saving (as an indicator of sustainability) and inputs are labour, physical capital and life satisfaction. We use data envelopment analysis (DEA), a linear programming technique, to compute optimal weights to aggregate inputs and outputs to derive productivity indicators. Since the seminal paper by Charnes et al. (1978), the number of publications using DEA to assess efficiency/productivity has been on the rise (Emrouznejad and liang Yang (2018) counted more than 10,000 publications using DEA between 1978 and 2016). Sickles and Zelenyuk (2019) provide a comprehensive treatment of both economic theory of productivity and its measurement using DEA.

The evolution of the DEA framework can be divided in two periods (Liu et al., 2013). The first one, up to 1999, is mainly driven by methodological development. A notable example in this regard is the research on returns to scale (RTS) to better characterize the production process (see, for instance, Seiford and Zhu (1999)). A second example is the decomposition and interpretation of DEA-productivity indicators in terms of efficiency change and technical change (see, for instance, Arcelus and Arozena (1999)). Another important contribution belonging to the early period of DEA, and related to the present work, is the introduction of undesirable output (Färe et al., 1989), such as pollution, and the possibility for outputs/inputs to take negative values (see for example Cooper et al. (1999a)).

The second period, starting after 1999, sees a new set of methodological developments about inference for certain measures of point efficiency by using appropriate bootstrap techniques (see, for instance, Kneip et al. (2008), Kneip et al. (2011), and Simar and Wilson (2011)), or to compare groups mean (see, for instance, Kneip et al. (2015), Kneip et al. (2016), or Kneip et al. (2021) for Malmquist indexes). Simar and Zelenyuk (2020) provide a recent groundbreaking study on inference and DEA. But this second period is in particular noticeable for the investigation of productivity in specific industries, such as banks, health care, agriculture and farm, transportation, and education. Particularly relevant for our work is the use of DEA in sustainability studies. This line of research started to grow after 2008 thanks to methodological improvements of the early 2000s, namely the introduction of concepts such as bad output, and the possibility to deal with negative values (Zhou et al., 2018). In particular, the introduction of sustainability issues in DEA empirical analysis marks an important theoretical development, as it seeks to include qualitative aspects in the computation of productivity. It is also worth noticing that – independently from the framework, hypotheses, decomposition of productivity indicators, and topics under scrutiny – these studies have a point in common:

the preliminary selection of inputs and outputs. In fact, the vast majority of studies adopts an a-priori set of inputs and outputs based on heuristic decision-making or expert judgement. However, some authors introduced data-driven methods exploiting DEA models to select the set of relevant inputs and outputs based on optimality criteria (see, for instance, the recent works by Peyrache et al. (2020) and Toloo et al. (2021).

Present work sits at the intersection of these developments. From a qualitative point of view, we investigate the suitability of accounting for life satisfaction and sustainability in the assessment of the performance of economies. From a technical point of view, we build on optimal selection methods to choose relevant inputs and outputs. In particular, we use a test procedure developed by Toloo et al. (2021).

3 Method and data

3.1 The variable selection method

Productivity[4] is commonly defined as the ratio of goods and services produced (output volume) by the quantity of resources used in the production processes (volume of inputs). Then,

$$\text{Productivity} = \frac{\text{output volume}}{\text{volume of inputs}} = \frac{\sum_i r_i y_i}{\sum_j w_j x_j}. \tag{1}$$

The $y_i, i = 1, ..., s$ are the outputs, in cross country analysis it is usually total GDP in constant terms, and the $x_j, j = 1, ..., m$ are inputs – at minimum physical capital K (machinery and equipments), and labour L (people or hours worked). Productivity measures how efficiently inputs are used in the production process as well as technological developments. The ratio increases when output volume increases for a given value of inputs. Similarly, the ratio increases if the volume of inputs reduces for a given value of output volume. In our case, we add life satisfaction (WBO), and/or adjusted net savings (ANS) to the set of outputs; and life satisfaction (WBI) to the list of inputs. Our starting point is:

$$\text{Productivity} = \frac{r_{GDP}GDP + r_{WBO}WBO + r_{ANS}ANS}{w_K K + w_L L + w_{WBI}WBI}. \tag{2}$$

The problem with equation (2) is the computation of weights (r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI}). One could use prices or income shares as weights (e.g.

[4]In this document productivity refers to total factor productivity.

OECD (2001)), but prices/income shares for life satisfaction and adjusted net saving do not exist. This problem is not new and motivates the seminal work by Charnes et al. (1978). The authors overcome the issue by developing a linear program that can be solved using DEA. This technique provides optimal weights to aggregate outputs and inputs to get a productivity indicator.

When computing optimal weights, one of the two modelling hypotheses have to be made: either we consider that countries manage to reduce inputs to increase productivity for a given level of outputs (input approach). Or, we assume that for a given level of inputs countries try to increase the amount of outputs produced (output approach). For the sake of present work, we follow the output oriented model. The reason is that we are interested in assessing productivity as the ability to increase outputs *given* the level of inputs used. In other words, we do not consider the hypothesis that a country is willing to decrease the use of inputs, in particular of life satisfaction, for a given level of outputs (as it is assumed in input oriented models). This amounts to assume that countries seek to increase sustainability and life satisfaction. However, we recall that, by definition, inputs are resources which are under the management's control. Inputs can be increased or decreased at will: if it is easy to envisage that countries seek to increase life satisfaction, it is not as obvious to imagine a country that deliberately chooses to decrease it. In some circumstances, however, this may be the case. Think, for instance, of the famous quote by Winston Churchill during the Second World War: "I have nothing to offer but blood, toil, tears and sweat". This is an example of a country asking sacrifices to the population during adversities or economic downturns. Arguably, however, this is not often the case. Therefore, we choose the output oriented approach and we assume that decreasing the use of inputs is not a favoured policy option. The output oriented model is the following:

$$
\begin{aligned}
&\max_{\lambda_j} \phi_0 \\
&\sum_j \lambda_j K_j \leq K_0 \\
&\sum_j \lambda_j L_j \leq L_0 \\
&\sum_j \lambda_j WBI_j \leq WBI_0 \\
&\sum_j \lambda_j GDP_j \geq \phi_0 GDP_0 \\
&\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0 \\
&\sum_j \lambda_j ANS_j \geq \phi_0 ANS_0 \\
&\lambda_j \geq 0
\end{aligned}
\tag{3}
$$

Appendix A shows the steps to go from equation (2) to model (3). This representation is useful to illustrate how we proceed to check whether life satisfaction is an input, output or both, and adjusted net savings belongs to the set of outputs. We adopt the procedure by Toloo et al. (2021) (Peyrache et al. (2020) propose a related approach). We re-write the model (3) as follows:

$$\max_{\lambda_j, d_{WBI}, d_{WBO}, d_{ANS}} \phi_0$$

$$\sum_j \lambda_j K_j \leq K_0$$

$$\sum_j \lambda_j L_j \leq L_0$$

$$\sum_j \lambda_j WBI_j \leq WBI_0 + M(1 - d_{WBI}) \tag{4}$$

$$\sum_j \lambda_j GDP_j \geq \phi_0 GDP_0$$

$$\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0 - M(1 - d_{WBO}) \tag{5}$$

$$\sum_j \lambda_j ANS_j \geq \phi_0 ANS_0 - M(1 - d_{ANS}) \tag{6}$$

$$d_{WBI} + d_{WBO} + d_{ANS} \leq k^{sup} \tag{7}$$

$$d_{WBI} + d_{WBO} + d_{ANS} \geq k_{inf} \tag{8}$$

$$\lambda_j \geq 0, \sum_j \lambda_j = 1, (d_{WBI}, d_{WBO}, d_{ANS}) \in \{0, 1\}^3.$$

In this model,M is a large positive number. Assume, for example, that $d_{WBO} = 1$ then constraint (5) becomes $\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0$, WBO contributes to the computation of productivity, and life satisfaction is an output. Conversely, if $d_{WBO} = 0$ the constraint becomes $\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0 - M$. As M is large, then the constraint is never binding ($\phi_0 WBO_0 - M < 0, \forall \phi_0$, M large enough) and life satisfaction does not contribute to productivity assessment. The same reasoning holds for other variables. Trivially, if $d_{WBI} = d_{WBO} = d_{ANS} = 1$ the model is equivalent to model (3).

Another important aspect of the model is the introduction of constraints (7) and (8). If $k_{inf} = 1$ then we impose to select at least one of the extra variables (WBI, WBO or ANS). If $k_{inf} = 1$ and $k^{sup} = 1$ then we want to have only one extra variable selected. If $k_{inf} = 1$ and $k^{sup} = 3$ then we can have from one to three extra variables in the computation of productivity.

In this framework, the status of life satisfaction and adjusted net savings as inputs and/or outputs is country and time specific. In principle, we could impose the set of inputs and/or outputs to be the same for all countries. It would suffice to stack the model across countries and/or time. However, we chose to have no a priori, and to use a specification that allows the status of life satisfaction and adjusted net savings to change over time and across countries. In other words, our model allows life satisfaction to be an input (output) for all countries at the same time, and/or for all years. The same holds for adjusted net savings. As explained by Toloo et al. (2021), the input and output oriented models can lead to select different variables. Toloo et al. (2021) propose a model that integrates both orientations in a single model.Again, we follow the output oriented approach as we consider the case of decreasing well-being as an input not a policy option.

A second important assumption concerns returns to scale. The model above assumes variable returns to scale, as clarified by the constraint $\sum_j \lambda_j = 1$. However, Toloo et al. (2021) documented that the same procedure holds also under the assumption of constant returns to scale (CRS). For our purposes, we assume CRS as it is a good benchmark to assess productivity for countries. In addition, in the case of CRS, productivity measurements yield similar results under the input and the output oriented models. A final important point for our work relates to the computation of Malmquist productivity index. Some authors (e.g. Kerstens and Van de Woestyne (2014)) claim that the Malmquist productivity index has no total factor productivity (TFP) interpretation in general, and argue in favor of the HicksMoorsteen index. An advantage of choosing CRS is that the HicksMoorsteen index collapses to the usual Malmquist index, thus overcoming the disputes over the most appropriate measure of TFP. At worst, CRS model is conventionally regarded as the best discriminating DEA model then a relevent benchmark (Podinovski et al. (2014)). Last, in this document, we have opted for DEA but it would have also been possible to use stochastic frontier analysis (SFA). In this case, the idea is to follow a model selection approach between nested models for example in the line of work of Lai and Huang (2010).

3.2 Variables used to assess productivity

We retrieve measures of output (GDP) and inputs (capital and labor) from the Penn World Tables, version 10 (Feenstra et al., 2015). The sample includes 23 European countries (Austria, Belgium, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Ireland, Italy, Lithuania, Luxembourg, Netherlands, Poland, Slovakia, Slovenia, Spain, Sweden, Turkey, United Kingdom).

Adjusted net savings is computed by the World Bank. Adjusted Net Saving is the national saving minus fixed capital consumption plus education expenditure minus depletion of natural resources and minus damages from CO2 emissions and particulate emissions. Adjusted net savings is a standard indicator of (weak) sustainability[5]. Our data cover the period from 2005 to 2018 because of data availability about life satisfaction. Moreover, for the sake of simplicity, we select countries for which adjusted net saving is positive.[6]

We draw data on life satisfaction from the Eurobarometer (2005 - 2018). Eurobarometer is the polling instrument of the European Union, and it is used to regularly monitor the state of public opinion in Europe. It covers issues related to the European Union, as well as attitudes on subjects of political or social nature [7]. For instance, during the interview, people are asked to reply to the following question: "On the whole, are you very satisfied, fairly satisfied, not very satisfied, or not at all satisfied with the life you lead?". This is a typical wording used to monitor respondent's satisfaction with life. For the purposes of present study, we use the share of people, by country and year, declaring to be very satisfied with the life they lead.

A characteristic feature of our work is the simultaneous introduction of life satisfaction in the set of inputs (WBI) and in the set of outputs (WBO) of production. If WBI is measured as WBO then we would have a conflict between constraint (4) and (5). We overcome this difficulty thanks to a feature of the Eurobarometer. The survey is usually administered twice per year. For each year, we have two measurements of life satisfaction: one around August, and one in January. This gives us access to two temporally distinct measurements of life satisfaction. Specifically, we measure WBI as the share of people that are very satisfied with their life (as observed in the August surveys) multiplied by hours worked. Thus, WBI is the number of hours worked by the share of very satisfied people. Formally:

$$WBI = (\text{share of people very satisfied with their life}) \cdot \text{hours worked}_{\text{t}} \tag{9}$$

This amounts to treating life satisfaction as a multiplier on work force: the higher the share of people satisfied with their lives, the larger the positive effect on labor. This modelling approach is similar to the one adopted by Barro

[5] https://www.un.org/esa/sustdev/natlinfo/indicators/methodology_sheets/econ_development/adjusted_net_saving.pdf presents the indicator. Considering ANS instead of CO2, allows to analyse a broader concept of sustainability and not just CO2 damages. In any case, it is also possible to introduce CO2 (only) as a bad output as proposed by Jeon and Sickles (2004).

[6] As a remark, adjusted net savings can be negative, in this case a specific DEA model has to be used, for example Cooper et al. (1999b). However, the main idea behind the variable selection procedure remains the same. In this document we restrict ourselves to the case of positive adjusted net savings.

[7] (https://europa.eu/eurobarometer/about))

and Lee (1994) regarding educational attainment, or by Botev et al. (2019) for human capital. Let δ_j be the share of people very satisfied with their life in country j, then the *total employment* input is $(1+\delta_j)\dot{hours}_j = \Omega_j \dot{hours}_j$. The effect of life satisfaction is reflected in the effective labour input as in the model by Lucas (1988). It would have been interesting to use job satisfaction instead but we are constrained by data availability.

As for WBO, we assume that Governments, to a certain extent, act as social benevolent planners who foster the production of more goods and services to satisfy a growing set of needs thus, ultimately, improving people's lives. In present work, this amounts to assume that countries seek to maximise the share of the population that is very satisfied with their life. From this point of view we are consistent with the idea of the benevolent social planner in theories of optimal growth model. WBO is based on life satisfaction measured in the month of January of each year, and it is defined as follows:

$$WBO = (\text{share of people very satisfied with their life}) \cdot \text{population}_\text{t} \tag{10}$$

we emphasize that WBI and WBO are observed at two different time periods: WBI relates to life satisfaction declared in the month of August at time t and it is multiplied by hours worked; WBO is based on the life satisfaction reported in January at time $t+1$, and it is multiplied by population[8].

Our hypotheses are:

1. Life satisfaction in productivity measurement is
   (a) an input only: $d_{WBI} = 1$ and $d_{WBO} = 0$ and:
      i. Adjusted net saving is an output $d_{ANS} = 1$ or,
      ii. Adjusted net saving is not an output $d_{ANS} = 0$.
   (b) an output only: $d_{WBI} = 0$ and $d_{WBO} = 1$ and:
      i. Adjusted net saving is an output $d_{ANS} = 1$ or,
      ii. Adjusted net saving is not an output $d_{ANS} = 0$.
   (c) an input and an output: $d_{WBI} = 1$ and $d_{WBO} = 1$ and:
      i. Adjusted net saving is an output $d_{ANS} = 1$ or,
      ii. Adjusted net saving is not an output $d_{ANS} = 0$.
   (d) not an input and not an output: $d_{WBI} = 0$ and $d_{WBO} = 0$ and:

[8] Many micro-econometric studies treat subjective well-being measures as cardinal, and some scholars warned that this approach may lead to biased results (Kaiser and Vendrik, 2020). However, this does not apply to our work: present analysis is at country level, and we use the proportion of respondents declaring to be very satisfied with their life by country.

i. Adjusted net saving is an output $d_{ANS} = 1$ or,

ii. Adjusted net saving is not an output $d_{ANS} = 0$.

4 Results

The results of the optimal selection method indicate that life satisfaction appears either as an input or as an output for almost all countries and all years considered (see table 1). The countries where life satisfaction is always or almost always an input are the Nordic countries: Denmark, Sweden, Finland; some western countries, such as Luxembourg, Ireland, Netherlands, United Kingdom; and Cyprus, Turkey and Poland. These countries are prevalently characterized by high levels of well-being. The countries where life satisfaction is an output are Eastern countries, such as Estonia, Hungary, Czech Republic, Slovakia and Lithuania, and some western countries: for example, Germany, Spain and France. OECD (2020) note that these three last countries are among the economies where the majority of the headline indicators composing the OECD Better Life Index index improved. Belgium and Slovenia are the only countries where adjusted net savings appear most of the time as an output. Interestingly, life satisfaction is never at the same time an input and an output of the production process, nor adjusted net savings and life satisfaction are concurrently outputs. Each year only one extra variable is selected.

In sum, the method for optimal selection of variables indicates that it is meaningful to correct traditional measures of productivity including life satisfaction among the inputs and outputs of production.

What makes life satisfaction an input or an output of the production process based on our data? To answer this question, we use classification tree, a data exploration tool that allows us to group similar observations. This technique is particularly useful to investigate the features of country-years (number of observations = 23 countries * 14 years = 322) when life satisfaction is an input or life satisfaction and adjusted net savings are outputs. The classification tree selects countries into groups based on the optimal values of the dichotomous variables $d_{WBI}, d_{WBO}, d_{ANS}$.

Figure 1 shows some of the partitions generated by the algorithm. We find that a significant number of country-years for which life satisfaction is an input are characterized by a large share of their population being very satisfied with their life (over 36%). This group includes countries such as: Denmark, Luxembourg, Netherlands, Sweden, United Kingdom and Poland. The latter is rather an exception: differently from the other countries, Poland exhibits a lower share of very satisfied people (between 11% and 36%), and a low level

Table 1: Share of times (in percentage) that WBI, WBO and ANS meaningfully contribute to the estimates of productivity. The share is computed over the pooled sample of countries-years.

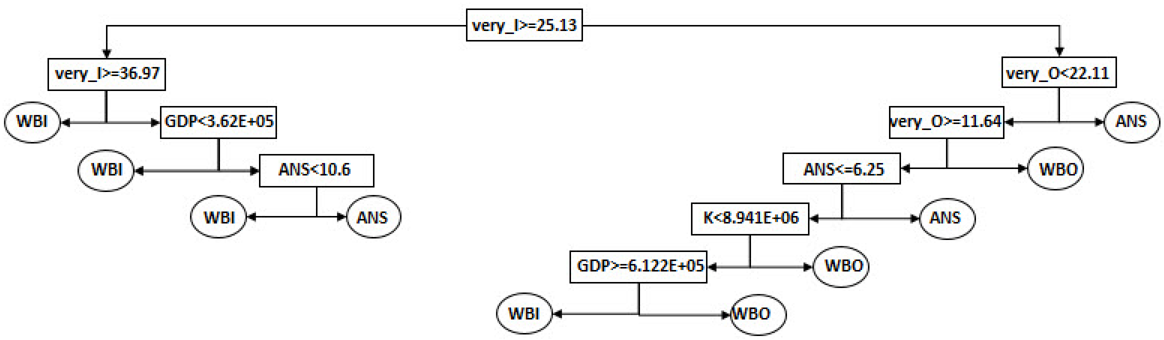
country	WBI	WBO	ANS	
Denmark	100	0	0	WBI only
Sweden	100	0	0	
Netherlands	100	0	0	
Ireland	100	0	0	
Poland	100	0	0	
United Kingdom	86	0	14	Mainly WBI
Finland	79	0	21	
Luxembourg	71	29	0	
Cyprus	71	7	21	
Turkey	57	7	36	
Estonia	0	100	0	WBO only
Hungary	0	100	0	
Italy	0	100	0	
France	0	93	7	Mainly WBO
Lithuania	14	86	0	
Czech Republic	0	86	14	
Slovakia	0	64	36	
Austria	0	64	36	
Spain	0	71	29	
Germany	21	43	36	
Croatia	21	36	43	Mainly ANS
Slovenia	21	7	71	
Belgium	14	7	79	

Note: authors' own computations on PWT v.10, and Eurobarometer data. WBI only: Well-being is an input all years, WBO only: Well-being is an output all years, Mainly WBI: Well-being is an input most of the years, Mainly WBO: Well-being is an output most of the years, Mainly ANS: ANS is an output most of the year.

of physical capital compared to its GDP.

Countries listing adjusted net savings as outputs are divided into two main groups: the first one is characterized by countries with a relatively large share of people very satisfied with their life, high GDP, and high adjusted net savings (this is the case of Belgium, for instance). The second group includes countries with an average share of people very satisfied with their life, or with a relatively high value of adjusted net savings. Slovenia and Turkey are examples of countries belonging to this group. For the remaining countries, mainly characterised by low shares of people very satisfied with their life, life satisfaction appears mainly an output of the production process.

Figure 1: Segment of a classification tree to group countries based on life satisfaction (input and output) and adjusted net savings.



Note: authors' own computations on PWT v.10, and Eurobarometer data. very_I: share of people very satisfied with their life (mid-year - input) very_O: share of people very satisfied with their life (beginning of year - output) K: capital, ANS: adjusted Net Saving, WBI well-being input, WBO well-being output. Left branch: condition is true. Right branch: condition is false.

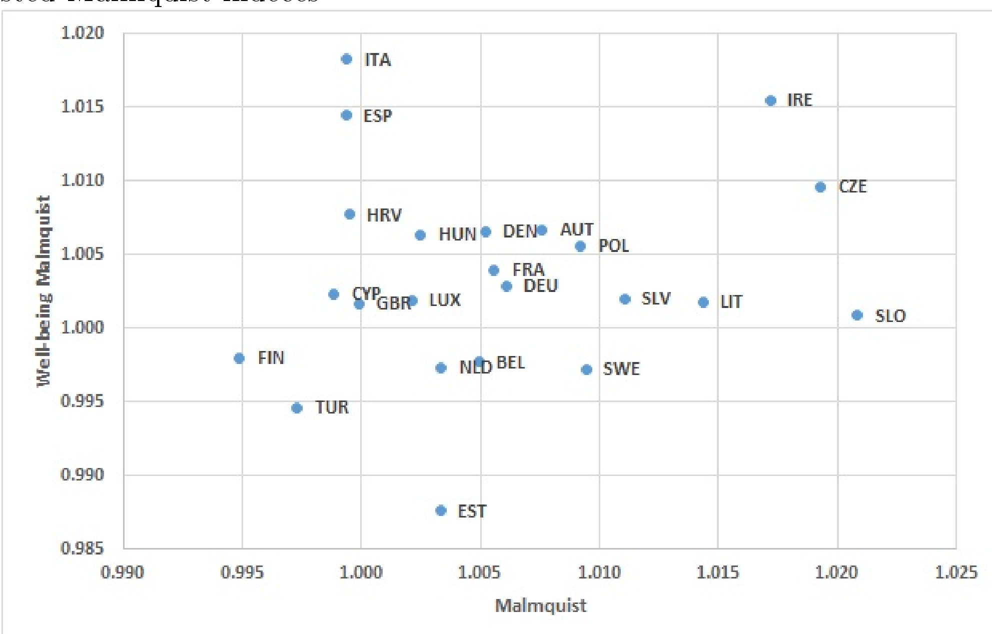
If it is meaningful to add life satisfaction among the inputs and outputs of production, how would such well-being adjusted productivity look like? This is the last step of our analysis: we compute well-being adjusted Malmquist productivity (see the vertical axis of figure 2), and we contrast it with traditional Malmquist productivity index (see the horizontal axis of figure 2). We recall that well-being adjusted productivity includes life satisfaction as an input and as an output, assumes constant returns to scale, and it is based on an output oriented method (see Grifell-Tatj and Lovell (1995) for a presentation of Malmquist TFP indexes).

We recall that DEA is a benchmarking exercise where countries having the best performance receive a score of 1 and are on the frontier. The lower the score is, the less efficient countries are. In our case, 4 countries are always on the frontier: Italy, Ireland, Poland, and Denmark. Laggard countries, with the lowest average performance, are Eastern European countries such as Slovenia (average score 0.75), Croatia (0,77), Czech Republic (0,77), Slovakia (0,85) or Lithuania (0,88). Luxembourg is an interesting case: it was on the frontier from 2005 to 2009 and then its score decreased constantly to reach a value of 0.79 – one of the least efficient countries in 2017.

Figure 2 indicates that, in general, if a country has a positive growth rate for TFP (Malmquist over unity) it has also a positive growth rate for well-being adjusted productivity. The two measures correlate quite well for some countries, such as Luxembourg. However, the association between the two measures is not statistically significant: some countries have a significantly lower well-being Malmquist than TFP Malmquist (Slovakia is a good example), whereas other countries, such as Italy or Spain, report almost no TFP growth, but large well-being adjusted Malmquist values. In other words, when we account for life satisfaction among the inputs and outputs of production, we find that some countries appear more efficient in transforming inputs into outputs than they usually are using Malmquist index. The Spearman's rho of similarity of rankings is 0.10, not statistically significant ($Prob > |t| = 0.6472$). Thus, we conclude that the two indexes provide significantly different information from each other. The top five countries in the well-being adjusted Malmquist ranking are Italy, Ireland, Spain Czech Republic and Croatia. The bottom five are: Belgium, Netherlands, Sweden, Turkey and Estonia.

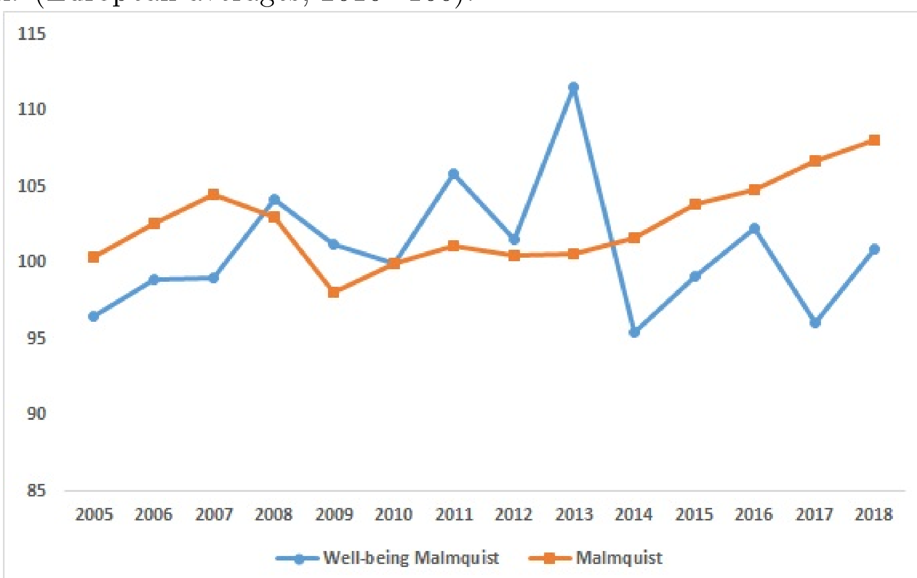
The comparison of European averages of the two indexes over time reveals that Malmquist TFP is less volatile than well-being adjusted Malmquist. Moreover, the trend of well-being adjusted Malmquist seems at odd with the trend of Malmquist index. We can distinguish two periods: the first one, from 2005 to 2009, is characterized by a positive trend for well-being adjusted Malmquist index, and a negative one for Malmquist index. The second period, from 2009 on-ward, is characterized by an un-interrupted growth of the Malmquist index, and flat (if not declining) well-being adjusted Malmquist index. The break in the trend of well-being adjusted Malmquist index between 2013 and 2014 appears as particularly striking.

Figure 2: Correlation between average Malmquist (TFP) and well-being adjusted Malmquist indeces



Note: Each indicator minus 1 is a growth rate. A value of 1 means a growth rate of 0.

Figure 3: Changes over time of Malmquist index and of the well-being adjusted index. (European averages, 2010=100).



Note: authors' own computations on PWT v.10, and Eurobarometer data.

5 Conclusion

Is it desirable and possible to build measures of productivity that account for people's well-being? Our answer, based on data from 23 European countries monitored over 14 years, is affirmative. It is desirable to build improved measures of productivity that keep into account the fact that economic activity, per se, is not strictly good or bad for quality of life and for the environment. From this point of view, much of previous elaboration focused on providing frameworks to integrate (mainly) environmental variables in traditional productivity measurements. It is also desirable because recent studies provided convincing evidence that people's well-being contributes to productivity, and that subjective well-being is not necessarily an outcome of production process. In 1968, Kennedy notoriously stated that GDP "measures everything in short, except that which makes life worthwhile". We also show that it is possible to integrate subjective well-being measures in traditional productivity computations, thus trying to go beyond the usual economic variables. Our answers are based on a data-driven approach for optimal selection of variables (Toloo et al., 2021).

Specifically, we check whether life satisfaction – a widely used, valid and reliable measure of subjective well-being – contributes meaningfully to productivity measures as an input and/or as an output, and that at the same time adjusted net savings – a proxy for sustainability – is an output of production. Results indicate that life satisfaction should be considered among the inputs and the outputs of production. Moreover, we found that life satisfaction is likely an input in countries where the share of people satisfied with their life is high (above 36%). Conversely, life satisfaction is likely an output in countries where the share of people satisfied with their life is low.

We used the results of our analysis to compute well-being adjusted Malmquist productivity indexes, and we contrasted the new variable with usual Malmquist indexes. Evidence indicates that the ranking of countries based on well-being adjusted Malmquist indexes is significantly different from the one derived from the usual Malmquist index. The correlation coefficient of the Spearman's rank test is 0.10, not statistically different from zero. Finally, the changes over time of the European averages of the two indexes indicate that well-being adjusted Malmquist indexes are more volatile than the usual indexes, and the two follow different trajectories: the first period, between 2005 and 2008, shows a positive trend which continues until 2013 when it reverts. The well-being adjusted Malmquist index indicates a remarkable break in the series between 2013 and 2014. The Malmquist index, on the contrary, follows a positive trend from 2009 on-ward.

Our work is not free from limitations and caveats. As we do not detect life satisfaction as an output and simultaneously as an input, we do not definitely solve the issue about what is the best indicator to compare countries. However, our results indicate that life satisfaction should be taken on board. We do so by including it among the inputs and the outputs of production. Furthermore, productivity indicators based on DEA are usually decomposed into efficiency and technical change. In our case, it is challenging to conceptualize the meaning of technical change for well-being adjusted productivity indicators. Perhaps, new wordings, such as societal progress, should be introduced to speak about technical change in relation to well-being. We also point out that high productivity growth rates can coexist with deteriorating economic and social conditions. As the efficient frontier is a relative benchmark, an inefficient country may experience productivity growth if best performers lose efficiency. Under these circumstances, productivity growth does not reflect economic and social progress.

It is also important to clarify some caveats related to the application of efficiency to subjective well-being. First, we stress that the underlying idea of efficiency indicators is that improvements can be attained when less inputs are used to produce at least the same level of output. In other words, from the efficiency point of view, if subjective well-being is an input, it may be optimal to reduce it. This option may not be socially desirable or acceptable. Thus, our productivity measure implicitly assumes that Governments are benevolent and interested in expanding well-being. Another caveat has to do with the substitutability of outputs. Assume that the computation of productivity indexes uses subjective well-being, adjusted net saving, and GDP as outputs. In this circumstances, the level of productivity could remain the same if the combination of outputs (aggregate value) remains unchanged. This is equivalent to saying that GDP, sustainability, and subjective well-being may be substitutable. This is the same critic that is often addressed to indicators of sustainability drawing a distinction between weak and strong sustainability. In this case, our well-being adjusted measure of productivity is a weak-productive-well-being indicator.

With these limits and caveats in mind, we believe that our contribution provides a sensible framework to include direct measures of utility (subjective well-being) in traditional productivity computations. This framework is in its infancy and could be refined in various ways. For instance, it would be interesting to check the robustness of our findings in presence of longer time-series and a larger sample of countries, not just European ones. It would also be desirable to check to what extent our results are robust to the use of objective measures of well-being, such as mental health, cortisol levels and other bio-physical markers, or drug consumption. Unfortunately, to the best of our knowledge, objective measures of well-being are not widely available or comparable across countries and over time. Another interesting approach would be to consider the creation of well-being as a several step process using network DEA. In a first step, GDP and adjusted net savings result from the use of economic resources such as labour and capital. Then, as a second step, GDP and adjusted net savings generate well-being. Last, there is a point that we do not investigate, the computation of shadow prices associated to well-being variables. As explained by Forsund (2018), it would help to assess the marginal productivity of input x_j in terms of the output of type y_i but also the marginal rate of transformation between output y_i and $y_{i'}$, and, the marginal rate of substitution between input x_j and $x_{j'}$. It would certainly offer interesting insights on the contribution of well-being to productivity.

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A Appendix

Our starting point is:

$$\text{Productivity} = \frac{r_{GDP}GDP + r_{WBO}WBO + r_{ANS}ANS}{w_K K + w_L L + w_{WBI}WBI}. \qquad (11)$$

The problem with equation (11) is the computation of weights (r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI}). Data Envelopment Analysis is a convenient framework to find optimal values for the weights without additional information (in particular if prices are not available). The idea is to compute optimal weights so that the ratio of equation (11), for a country labelled 0, is as large as possible, and the same ratios for all other countries are positive and below 1. As a consequence, countries with the highest ratios will have an optimal value of 1, and the closest to zero the ratio is, the lowest the efficiency is. This method allows to benchmark countries with respect to the most efficient ones. Formally, we have the following fractional program:

$$\max_{(r_{GDP},r_{WBO},r_{ANS},w_K,w_L,w_{WBI})} \frac{r_{GDP}GDP_0 + r_{WBO}WBO_0 + r_{ANS}ANS_0}{w_K K_0 + w_L L_0 + w_{WBI}WBI_0}$$

$$s.t. \frac{r_{GDP}GDP_j + r_{WBO}WBO_j + r_{ANS}ANS_j}{w_K K_j + w_L L_j + w_{WBI}WBI_j} \leq 1, j = 1, ..., N$$

$$r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI} \geq 0$$

This model can be converted into a linear program model, as follows: let $t = 1/(w_K K_j + w_L L_j + w_{WBI}WBI_j)$, then the previous fractional program becomes:

$$\max_{(r_{GDP},r_{WBO},r_{ANS},w_K,w_L,w_{WBI})} t(r_{GDP}GDP_0 + r_{WBO}WBO_0 + r_{ANS}ANS_0)$$
$$t(w_K K_0 + w_L L_0 + w_{WBI}WBI_0) = 1$$
$$s.t.$$
$$t(r_{GDP}GDP_j + r_{WBO}WBO_j + r_{ANS}ANS_j)$$
$$- t(w_K K_j + w_L L_j + w_{WBI}WBI_j) \leq 0, j = 1, ..., N$$
$$r_{GDP}, r_{WBO}, r_{ANS}, w_K, w_L, w_{WBI} \geq 0$$
$$t \geq 0$$

Changing notation, $tr_y = u_y$ and $tw_x = v_x$ ($y \in \{GDP, WBO, ANS\}, x \in \{K, L, WBI\}$), then:

$$\max_{(u_{GDP},u_{WBO},u_{ANS},v_K,v_L,v_{WBI})} u_{GDP}GDP_0 + u_{WBO}WBO_0 + u_{ANS}ANS_0$$
$$v_K K_0 + v_L L_0 + v_{WBI}WBI_0 = 1$$
$$s.t.$$
$$(u_{GDP}GDP_j + u_{WBO}WBO_j + u_{ANS}ANS_j)$$
$$- (v_K K_j + v_L L_j + v_{WBI}WBI_j) \leq 0, j = 1, ..., N$$
$$u_{GDP}, u_{WBO}, u_{ANS}, v_K, v_L, v_{WBI} \geq 0$$

Last, this linear program has a dual representation:

$$\min_{\lambda_j} \theta_0$$
$$\sum_j \lambda_j K_j \leq \theta_0 K_0$$
$$\sum_j \lambda_j L_j \leq \theta_0 L_0$$
$$\sum_j \lambda_j WBI_j \leq \theta_0 WBI_0$$
$$\sum_j \lambda_j GDP_j \geq GDP_0$$
$$\sum_j \lambda_j WBO_j \geq WBO_0$$
$$\sum_j \lambda_j ANS_j \geq ANS_0$$
$$\lambda_j \geq 0$$

In this model any improvement in productivity can only be obtained by decreasing the use of inputs. In this case, we speak of an input oriented model. Alternatively, one may be interested in assessing to what extent outputs can be increased given the use of inputs. In this case, we refer to the output oriented model:

$$
\begin{aligned}
&\max_{\lambda_j} \phi_0 \\
&\sum_j \lambda_j K_j \leq K_0 \\
&\sum_j \lambda_j L_j \leq L_0 \\
&\sum_j \lambda_j WBI_j \leq WBI_0 \\
&\sum_j \lambda_j GDP_j \geq \phi_0 GDP_0 \\
&\sum_j \lambda_j WBO_j \geq \phi_0 WBO_0 \\
&\sum_j \lambda_j ANS_j \geq \phi_0 ANS_0 \\
&\lambda_j \geq 0
\end{aligned}
$$

This model is the starting point of the procedure developed by Toloo et al. (2021) to select variables.