Alternative Output, Input and Income Concepts for the Production Accounts

W. Erwin Diewert
University of British Columbia and UNSW Sydney

and

Kevin J. Fox*
UNSW Sydney

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Abstract

Definitions of output and input are key to studies of productivity analysis, as they are to the national accounts of countries. This paper systematically reviews alternative definitions at production unit and aggregate levels, illustrating the different perspectives that they provide on production and income, and making the case for their use in understanding different aspects of firm and country economic performance.

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* Corresponding author: Kevin J. Fox: School of Economics & CAER, UNSW Australia, Sydney 2052 (K.Fox@unsw.edu.au). We thank two anonymous referees, Paul Schreyer, Bert Balk and participants at the 2023 ESCoE Conference, King's College London, for their constructive comments.

1. Introduction

This paper takes a new look at the production accounts in the international System of National Accounts with emphasis on alternative measures of output and primary input, with implications for the resulting alternative measures of productivity. Perhaps more importantly, the paper considers alternative measures of *income* generated by the production sector of an economy.

As well as their central use in informing macroeconomic policy, national accounts data on inputs and outputs for countries are used extensively in the academic literature on productivity; see for example Solow (1957), Jorgenson and Griliches (1967), Diewert and Fox (1999) and Fernald and Inklaar (2020). They are also used in the literature on efficiency analysis; see for example Färe, Grosskopf, Norris and Zhang (1994) and Kumar and Russell (2002). Given their extensive use and broad acceptance as the authoritative source of information on economic performance, it is tempting to believe that all matters relating to national accounts have been settled by the international community. Yet the United Nations System of National Accounts (SNA), which provides guidance to countries on optimal practice, is periodically revised.[1] Hence, it seems worthwhile to suggest an accounting framework which would supplement the usual gross and net domestic product measures with a measure that would better measure the income generated by the production sector of a national economy.[2]

Here we start from some basic definitions which lead us to propose alternatives measures of output, input and income. In doing so, we stay within the current production boundaries of the SNA 2008. That is, our paper is not a contribution to the growing literature on

[1] At the time of writing, the current version is the SNA 2008 (United Nations 2009), with the next revision due to be released in 2025.

[2] It should be noted that our preferred measure of income generated by a production unit is very close to Balk's Net Value Added; see Balk (2010; S244) (2011; 503). See also Schreyer (2009; 43-51) on net income measures in the System of National Accounts. Alternative income concepts are extensively discussed in Stiglitz, Sen and Fitoussi (2009).

"Beyond GDP" concepts nor on "GDP and Beyond", but rather stays focussed on alternatives within the existing SNA production boundary.[3]

The model of production that we use in this paper is based on treating capital as both an input used and output produced by the production sector of an economy. This model of production was developed by the economist Hicks and the accountants Edwards and Bell as shown by the following two quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period

[3] The "Beyond GDP" literature typically focusses on ending the use of GDP in policy making in favour of alternatives measures of progress. The "GDP and Beyond" literature focusses retaining GDP but with possible extensions to better capture things that are important yet are not currently (well) measured in the national accounts, such as household work, consumption of free digital goods, or the use of the environment as an input. See Stiglitz, Sen and Fitoussi (2009), Coyle and Mitra-Kahn (2017), OECD (2018), Corrado, Fox, Goodridge, Haskel, Jona-Lasinio, Sichel and Westlake (2017) and Brynjolfsson, Collis, Diewert, Eggers and Fox (2019).

"flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period). This model of production could be viewed as an *Austrian model of production* in honour of the Austrian economist Böhm-Bawerk (1891) who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.[4]

The rest of the paper is organised as follows. The next section introduces production accounting using a simplified context of a single production unit. Section 3 considers alternative net output, input and income concepts for a production unit, and section 4 provides additional discussion about our accounting framework. Section 5 considers corresponding economy wide measures with multiple types of capital and section 6 concludes.

2. Production Unit Accounting: The Hicks and Edwards and Bell framework

In order to simplify the notation, we consider a very simple model of production in this section for a *single production unit* that produces or uses only six types of goods and services during an accounting period t. A production unit could be a firm, a division of a firm or what national income accountants call an establishment. The establishment must be able to provide period by period accounting information about periodic revenues and costs as well as balance sheet information on the status of its asset holdings at the end of each accounting period.

Equation (1) below defines the production unit's *pure profits* in period t, Π^t, using the Hicks, Edwards and Bell approach to production theory:

$$\Pi^t \equiv P_Y^t\, Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t - P_L^t Q_L^t + P_K^t Q_K^t - (1+r^t) P_K^{t-1} Q_K^{t-1}. \qquad (1)$$

[4] This Austrian model of production was further developed by von Neumann (1937) and Malinvaud (1953) but these authors did not develop the user cost implications of the model. On the user cost implications of the Austrian model, see Hicks (1973; 27-35) and Diewert (1977; 108-111) (1980; 472-474). Balk (2010) (2011) used this neo-Austrian accounting framework.

The price and quantity variables appearing on the right hand side of (1) are defined as follows:

P_Y^t ≡ (unit value) price of output Y during period t;
Q_Y^t ≡ total quantity of output y produced during period t;
P_Z^t ≡ (unit value) price of intermediate input Z purchased during period t;
Q_Z^t ≡ total quantity purchased of intermediate input Z purchased during period t;
P_{IP}^t ≡ (unit value) price of one unit of an investment good *purchased* during period t;
Q_{IP}^t ≡ total number of units of the investment good *purchased* during period t;
P_L^t ≡ wage rate for one hour of labour used by the producer during period t;
Q_L^t ≡ total hours worked in period t by the type of labour under consideration;
P_K^t ≡ price of a unit of the capital stock held by the unit at the end of period t;
Q_K^t ≡ quantity of the capital stock held by the production unit at the end of period t;
P_K^{t-1} ≡ price of a unit of the capital stock held by the unit at the beginning of period t;
Q_K^{t-1} ≡ quantity of the capital stock held by the unit at the beginning of period t;
r^t ≡ period t cost of capital for the production unit.

Units of the total output Q_Y^t could be sold to domestic customers or could be exported. Later in the paper, this distinction will become important when we aggregate over producers but at present, we do not have to distinguish domestic sales from foreign sales. Similarly, units of the intermediate input and units of the investment good could be purchased from domestic suppliers or could be imported.[5]

We note that prices and quantities of output, intermediate input, purchased investment goods and labour can in principle be observed by the accountant. However, the quantity and price of the production unit's beginning and end of period capita stocks, Q_K^{t-1}, Q_K^t,

[5] If the Production Unit (PU) is producing the investment good as an output, then sales of these investment goods are included in $P_Y^t Q_Y^t$. However, for the PU that purchases the investment good, the purchases are recorded in the purchasing unit's $P_{IP}^t Q_{IP}^t$. Similarly, flow outputs of the PU under consideration that are purchased by other domestic units are recorded in the purchasing unit's $P_Z^t Q_Z^t$.

P_K^{t-1} and P_K^t, *typically cannot be observed* but must be estimated by the accountant. We will indicate how this can be done shortly. The production unit's period t *cost of capital* is denoted by r^t on the right hand side of (1). If the production unit purchased its beginning of period t capital stock and financed this purchase by issuing a one period bond at the interest rate r^{t^*} in the amount equal to $P_K^{t-1}Q_K^{t-1}$, then r^t in definition (1) would equal the observed bond interest rate r^{t^*}.[6] However, in general, since a firm's holdings of beginning of the period assets are financed by a mixture of debt and equity capital, a firm's weighted cost of capital must be estimated by the national income accountant since there is no unambiguous estimate for the equity portion of a firm's financial capital.

Standard firm accounting does not allow for a deduction for the cost of equity capital[7] but following Hicks' (1946) intertemporal theory of the firm, it is clear that future cash flows should be discounted by an appropriate interest rate or cost of capital in order to make future cash flows comparable to present cash. Accounting conventions suggest that current period flows should be cumulated over the accounting period and "realized" at the end of the accounting period.[8] Thus the discounted pure profits of the production unit for period t are equal to minus the beginning of the period cost of the capital stock, $-P_K^{t-1}Q_K^{t-1}$, plus the period t *discounted cash flow* of firm revenues minus firm expenditures on flow inputs and market purchases of investment goods, $(1+r^t)^{-1}(P_Y^t\,Q_Y^t - P_Z^tQ_Z^t - P_{IP}^tQ_{IP}^t - P_L^tQ_L^t)$, plus the discounted end of period value of the production unit's capital stock, $(1+r^t)^{-1}P_K^tQ_K^t$. But if we measure profits from the *perspective of the end of period t*, then the resulting "anti-discounted" profits are equal to $(1+r^t)P_K^{t-1}Q_K^{t-1}$ plus cash flow plus the value of the capital stock at the end of period t, which is equal to pure profits Π^t defined by (1).

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[6] See Diewert (2014) for a more complete accounting model that deals with the financing of the initial capital stock and other financial transactions using the Hicksian accounting framework.
[7] This accounting convention dates back to Garske and Fells (1893). For a discussion of this convention, see Anthony (1973). Diewert and Fox (1999) attributed some of the fall in the worldwide fall in Total Factor Productivity during the 1970s to the problems associated with measuring income using historical cost accounting when inflation is high.
[8] "This [convention] accords with the assumption conventional in discrete compounding that flows occur at the end of each period." K.V. Peasnell (1981; 56).

At this point, we need to make some assumptions about investments, depreciation and capital stocks. The first point to note is that, in general, investment goods could be purchased or they could be manufactured by the production unit. Thus we have defined P_{IP}^t and Q_{IP}^t as the period t price and quantity of purchased investment goods. However, the production unit may also produce units of the investment good internally for its own use. Thus define $Q_{II}^t > 0$ as the amount of internally produced investment (or own-account investment) and P_{II}^t as the imputed price for a unit of this internally produced investment.[9] Define period t *total investment* as the sum of purchased investment, Q_{IP}^t, plus internally produced investment, Q_{II}^t:

$$Q_I^t = Q_{IP}^t + Q_{II}^t \qquad (2)$$

Our next assumption relates period t total investment to the beginning and end of period t capital stocks held by the unit; i.e., we assume that the following equation holds:

$$Q_K^t = (1 - \delta^t)Q_K^{t-1} + Q_I^t \qquad (3)$$

where δ^t is the *period t geometric depreciation rate* that is applied to the production unit's beginning of the period capital stock Q_K^{t-1} in order to obtain the number of constant quality units of the initial capital stock at the end of period t that are equivalent to new units of the capital stock.[10]

The price of a new unit of the capital stock at the beginning of period t, P_K^{t-1}, should be equal to the price of a new investment good at the *beginning* of period t. Note that this

[9] If $Q_{II}^t = 0$, there is no need to impute P_{II}^t. If $Q_{II}^t > 0$, then define P_{II}^t as the average cost of producing the internally manufactured investment goods. Typically, Q_{II}^t will be a small amount of total investment. If firms make very large infrastructure investments such as building pipelines or new natural gas liquefaction plants, then internally produced investments become important.

[10] The geometric model of depreciation was used by Jorgenson and Griliches (1967) in their classic study of the Total Factor Productivity of the U.S. economy. For additional materials on the geometric model of depreciation, see Jorgenson (1989) (1996a) (1996b) and Schreyer (2001) (2009). Schreyer (2009) and Balk (2011) both introduce a modification of the classical geometric depreciation model by assuming that this period's investment adds to the productive capital stock at the midpoint of the present period instead of at the end of the current period. This is a reasonable assumption but implementing it leads to extra complications in that we need to construct separate user costs for new investments and the depreciated capital stocks at the end of the accounting period. Also deferring depreciation of newly purchased capital stocks until the period after their purchase is consistent with accounting conventions; see Peasnell (1981).

beginning of the period price is not necessarily equal to the period t market price of the investment good, P_{IP}^t, since P_{IP}^t price represents the *average price* of the investment good over the entire duration of period t. Similarly, the price of a new unit of the capital stock at the end of period t, P_K^t, is not necessarily equal to P_{IP}^t. If inflation is low, then P_K^t could be approximated by P_{IP}^t. If general inflation is high during period t, then P_K^t could be approximated by $(½)P_{IP}^{t-1} + (½)P_{IP}^t$.[11] More generally, one could argue that in a situation where asset prices are very volatile, instead of using the price of an investment good at the beginning and end of a period, one should use a longer run *smoothed investment price* for P_K^t that captures the *trend* in the price of a new unit of a particular capital stock component. Typically firms do not actually sell their capital stocks; they hold units of their capital stock until they are completely worn out. However, the owners of firms are interested in end of period values for the capital stocks held by the firm because there is always the option of selling these capital stocks. If asset prices are very volatile, using a smoothed estimate for the current values of capital stock components may give investors a more realistic picture of the current opportunity cost of holding the existing capital stocks in the production unit rather than using an estimated current value which is subject to large fluctuations.

In any case, we assume that the national income accountant has estimates available for the beginning and end of period t prices of a new unit of the capital stock. These prices can be used to define the *period t asset inflation rat*e i^t using the following equation:

$$1 + i^t = P_K^t / P_K^{t-1}. \quad (4)$$

Thus $P_K^t = (1 + i^t)P_K^{t-1}$. Now use (4) to eliminate P_K^t and use (3) to eliminate Q_K^t from definition (1). This allows us to express period t pure profits Π^t for the production unit as follows:

[11] Commercial accounting "solves" this capital stock valuation problem by using historical cost accounting which simply carries forward the initial purchase value of a capital stock and applies a suitable depreciation rate to this initial value without making any adjustment for price change. See Ijiri (1979) for a defense of historical cost accounting.

$$\begin{aligned}\Pi^t &= P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t - P_L^t Q_L^t \\ &\quad +(1+i^t)P_K^{t-1}[(1-\delta^t)Q_K^{t-1} + Q_I^t] - (1+r^t)P_K^{t-1}Q_K^{t-1} \\ &= P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t Q_I^t - P_L^t Q_L^t - U^t Q_K^{t-1}.\end{aligned} \tag{5}$$

The *period t user cost of capital* U^t which makes its appearance in the second line of (5) is defined as follows:[12]

$$\begin{aligned}U^t &= [(1+r^t) - (1+i^t)(1-\delta^t)]P_K^{t-1} \\ &= [r^t - i^t + (1+i^t)\delta^t]P_K^{t-1}.\end{aligned} \tag{6}$$

Thus the user cost of capital consists of three terms: the interest rate term $r^t P_K^{t-1}$, less an asset price inflation term $-iP_K^{t-1}$, plus a depreciation term valued at the end of period price of a new asset, $(1+i^t)\delta^t P_K^{t-1} = \delta^t P_K^t$.[13]

Note that the treatment of investment in expression (5) is not conventional: see the terms $-P_{IP}^t Q_{IP}^t + P_K^t Q_I^t$ which add to profits the value of *total investment* Q_I^t valued at the end of period price of a unit of capital, P_K^t, and subtract the value of *purchased investment* valued at market prices, $-P_{IP}^t Q_{IP}^t$. The remaining terms in (5) are conventional: $P_Y^t Q_Y^t - P_Z^t Q_Z^t$ is equal to revenues less payments for intermediate inputs, or value added, and $P_L^t Q_L^t + U^t Q_K^{t-1}$ is primary input cost made up of labour cost, $P_L^t Q_L^t$, and capital services cost, $U^t Q_K^{t-1}$.

It should be noted that a conventional economic treatment of firm accounting would not measure profits according to definition (1) or its special case (5) which was derived from (1) using assumptions (2)-(4). Conventional economic accounting would immediately

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[12] Babbage (1835; 287) described the user cost idea in words and Walras (1954; 268-269) developed an explicit user cost formula (in 1874) as did the industrial engineer Church (1901; 907-908). Alternative derivations of a user cost formula may be found in Jorgenson (1963) (1989) (1996b), Griliches (1963; 120), Christensen and Jorgenson (1969; 302), Diewert (1974; 504) and Diewert and Lawrence (2000; 276).

[13] If the asset is a land or structure asset, then the use of this input may also be subject to a property tax. If the period t property tax rate τ^t is a percentage of the beginning of the period value of the asset, then the user cost becomes $[r^t - i^t + (1+i^t)\delta^t + \tau^t]P_K^{t-1}$.

capitalize all investments and define *conventional period t pure profits* of the production unit, Π^{t^*}, as follows:

$$\Pi^{t^*} \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_L^t\, Q_L^t - U^t Q_K^{t-1}. \qquad (7)$$

However, Π^{t^*}defined by (7) will equal Π^tdefined by (1) or (5) *if* the end of period t price of capital, P_K^t, is set equal to the period t average price of market purchased investments, P_{IP}^t, *and if* there are no internally produced investment goods so that total investment, Q_I^t, equals purchased investment, Q_{IP}^t.

In the following section, we will look at alternative output and input measures that could be constructed using our Hicksian measurement framework.

3. Alternative Domestic Net Output, Input and Income Concepts

Period t Gross Domestic Input or Income generated by the production unit, GDI^t, can be defined as the value of labour services $P_L^t\, Q_L^t$ plus the value of capital services $U^t Q_K^{t-1}$ plus the value of pure profits Π^t:

$$GDI^t \equiv P_L^t\, Q_L^t + U^t Q_K^{t-1} + \Pi^t \qquad (8)$$

To get the measure of production unit output that corresponds to the income measure defined by (8), replace Π^t in (8) by the right hand side of (5). Period t *Gross Domestic Output*, GDO^t, is then defined as follows:

$$\begin{aligned} GDO^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t Q_I^t \qquad (9) \\ &= CVA^t + P_K^t Q_I^t \\ &= GDI^t \end{aligned}$$

where period t *Comprehensive Value Added* of the production unit, CVA^t, is defined as *Regular Value Added*, $VA^t \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t$, less market expenditures on the investment good, $P_{IP}^t Q_{IP}^t$.[14] Thus period t $CVA^t$is defined as:

$$\begin{aligned} CVA^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t \\ &= VA^t - P_{IP}^t Q_{IP}^t \end{aligned} \quad (10)$$

Suppose the following conditions hold:

$$P_K^t = P_{IP}^t \, ; Q_I^t = Q_{IP}^t \quad (11)$$

Then it can be seen that our measure of gross output, GDO^t, is equal to Regular Value added, VA^t.

The problem with the gross income measure, GDI^t defined by (8) is that it includes the value of depreciation as a component of income. But depreciation is not a component of income that can be spent on the purchase of consumer goods and services. Thus the depreciation component of user cost should be removed as a source of income and transferred to the net output accounts; i.e., depreciation should be treated as deduction from production unit revenues and be treated as a type of intertemporal intermediate input.[15] The period t value of depreciation (valued at end of period prices of the capital stock) is $P_K^t \delta^t Q_K^{t-1} = (1 + i^t)\delta^t P_K^{t-1} Q_K^{t-1}$. Subtract this term from period t Gross Domestic Income to define the period t *Net Domestic Income*, NDI^t, generated by the production unit:

[14] The production unit could be producing units of the capital stock and this production would be included in the definition of a firm's regular value added. However, purchases of units of the capital stock are not included in regular value added because the cost of purchased investment goods is capitalized and depreciated over time using normal accounting procedures. Comprehensive Value Added allows revenues from sales of the investment good and costs from purchases of the investment good to enter the net output aggregate.

[15] See Hicks (1946; 174) (1973; 155), Samuelson (1961) and Balk (2010) (2011) on alternative definitions of income and on the treatment of depreciation. See also Schreyer (2009; 43-51) and Stiglitz, Sen and Fitoussi (2009) on net income measures in the System of National Accounts.

$$NDI^t \equiv GDI^t - (1+i^t)\delta^t P_K^{t-1} Q_K^{t-1} \quad (12)$$
$$= P_L^t Q_L^t + [r^t - i^t] P_K^{t-1} Q_K^{t-1} + \Pi^t \text{ using (8) and (6).}$$

In order to obtain the output measure NDO^t that matches up with the net income measure NDI^t defined by (12), substitute the right hand side of (5) to eliminate $\Pi^t$from the second line in (12). We obtain the following expression for the *Net Domestic Output* NDO^t produced by the production unit during period t:

$$NDO^t \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] \quad (13)$$
$$= CVA^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] \text{ using definition (10)}$$
$$= CVA^t + P_K^t [Q_K^t - Q_K^{t-1}] \text{ using (3)}$$
$$= NDI^t.$$

The second line of (13) tells us that period t Net Domestic Output is equal to the production unit's Comprehensive Value Added, CVA^t, plus the production unit's period t gross investment, Q_I^t, less period t depreciation of the starting capital stock, $\delta^t Q_K^{t-1}$, valued at the end of period capital stock price, P_K^t. Note that $Q_I^t - \delta^t Q_K^{t-1} = Q_K^t - Q_K^{t-1}$ is period t net investment.

The measure of net output defined by (13) looks reasonable enough. It adds the value of net investment (valued at the end of period price for units of the capital stock) to a comprehensive measure of value added produced by the production unit during period t. Thus this net output measure is consistent with Pigou's (1941; 273-274) preference for an output measure that is consistent with maintaining the physical capital stock. However, the problem with the net output measures of output and income, $NDO^t$and NDI^t, is the fact that the income measure does not accurately measure the nominal income generated by the production unit over the period; NDI^t omits the capital gains (or losses) that accrue to the initial capital stock held by the production unit. Adding these capital gains to NDI^t leads to period t *Comprehensive Net Domestic Income* generated by the producer over period t, $CNDI^t$, defined as follows:

$$CNDI^t \equiv P_L^t Q_L^t + r^t P_K^{t-1} Q_K^{t-1} + \Pi^t \quad (14)$$
$$= NDI^t + i^t P_K^{t-1} Q_K^{t-1} \text{ using the second line in (12).}$$

The first line in (14) tells us comprehensive net income is equal to payments to labour $P_L^t Q_L^t$ plus interest and dividend payments to the owners of the production unit for tying up their capital for the period, $r^t P_K^{t-1} Q_K^{t-1}$, plus any pure profits Π^t that might have occurred.[16] The second line in (14) tells us that $CNDI^t$ is equal to NDI^tplus capital gains on the production unit's initial capital stock.

In order to determine the net output measure that matches up with the comprehensive measure of income defined by the first line in (14), we use the right hand side of (5) to eliminate Π^t from the right hand side of (14). We obtain the following expression for period t *Comprehensive Net Domestic Output*, $CNDO^t$ for the production unit:

$$CNDO^t \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] + i^t P_K^{t-1} Q_K^{t-1} \quad (15)$$
$$= CVA^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] + i^t P_K^{t-1} Q_K^{t-1} \quad \text{using (10)}$$
$$= CVA^t + P_K^t [Q_K^t - Q_K^{t-1}] + i^t P_K^{t-1} Q_K^{t-1} \quad \text{using (3)}$$
$$= CVA^t + P_K^t Q_K^t - (1 + i^t) P_K^{t-1} Q_K^{t-1} + i^t P_K^{t-1} Q_K^{t-1} \quad \text{using (4)}$$
$$= CVA^t + P_K^t Q_K^t - P_K^{t-1} Q_K^{t-1}$$
$$= NDO^t + i^t P_K^{t-1} Q_K^{t-1}$$

The second last line in (15) tells us that our comprehensive measure of net domestic product for the production unit $CNDO^t$ is equal to comprehensive value added, CVA^t, plus the value of the end of period capital stock, $P_K^t Q_K^t$, less the value of the beginning of the period capital stock, $P_K^{t-1} Q_K^{t-1}$. This is a very straightforward definition of net (nominal) output. On the other hand, the net domestic measure of output, NDO^t, is equal to CVA^t plus the net change in the capital stock evaluated at end of period prices, $P_K^t [Q_K^t - Q_K^{t-1}]$. The last line in (15) shows that $CNDO^t$ is equal to NDO^t plus asset appreciation $i^t P_K^{t-1} Q_K^{t-1}$ if the

[16] Rymes (1968) (1983) defined $r^t P_K^{t-1} Q_K^{t-1}$ as *waiting services* and advocated replacing the user cost of capital by waiting services. The term "waiting" can be traced back to Marshall (1920; 232): "And human nature being what it is, we are justified in speaking of the interest on capital as the reward of the sacrifice involved in the waiting for the enjoyment of material resources, because few people would save much without reward; just as we speak of wages as the reward of labour, because few people would work hard without reward".

asset inflation rate i^t is positive. If i^t is negative due to obsolescence or other reasons, then Comprehensive Net Domestic Output will be less than Net Domestic Output. Thus the comprehensive net income measure is a maintenance of financial capital approach to the measurement of income whereas the net income measure is a maintenance of real physical capital approach.

Having estimates of the nominal income generated by a production unit is not the end of the story. In order to evaluate the contributions of a production sector to the creation of income, it is useful to convert the nominal income measure into a real income measure. That is, the nominal measure of income can be divided by a consumer price index to convert nominal income flows into real income flows. We note that our suggested comprehensive measure of real income generated by a production unit (which is $CNDI^t$ deflated by a consumer price index) is exactly the income concept recommended by the accountant Sterling:

"It follows that the appropriate procedure is to (1) adjust the present statement to current values and (2) adjust the previous statement by a price index. It is important to recognize that *both* adjustments are necessary and that neither is a substitute for the other. Confusion on this point is widespread." Robert R. Sterling (1975; 51).

Sterling (1975; 50) termed his income concept *Price Level Adjusted Current Value Income*. Unfortunately, Sterling's income concept has not been widely endorsed in accounting circles due to difficulties in implementing it in an unambiguous manner. But conceptually, Sterling's income concept is consistent with our Comprehensive Net Domestic Product income concept that is deflated by a consumer price index.

Which income concept is "best"? The gross income concept clearly overstates sustainable consumption and so this concept can be dismissed. However, choosing between the physical and real financial maintenance perspectives is more problematical: reasonable economists could differ on this choice. The merits of the two perspectives were debated by Pigou and Hayek over 80 years ago. Pigou (1941; 273-274) favoured the maintenance of physical capital approach while Hayek (1941; 276-277) favoured the maintenance of real financial capital approach (the approach of Sterling). Hayek noted that obsolescence of a capital good[17] leads to a loss of income which is not captured in the maintenance of physical capital approach to income measurement but it is captured in the maintenance of financial capital approach. Moreover, the approach of Pigou does not capture the gains in income that are generated by increasing land prices. The amount of land could remain constant but increases in the price of business land that are greater than the change in the consumer price index should lead to an increase in the real income generated by a production unit but the physical approach neglects these real income gains. If a price increase in an asset is foreseen, then the revaluation term can be regarded as a positive contribution to the net revenues produced by the production unit under consideration; i.e., the unit "transports" the asset from a time when it is less valued to a time when it is more highly valued.

As Hicks (1946; 184) said in his income chapter: "What a tricky business this all is!"

Table 1 shows the relationship of the three alternative definitions of output and relationship of the three matching definitions of income or primary input, where $CVA^t \equiv P_Y^tQ_Y^t - P_Z^tQ_Z^t - P_{IP}^tQ_{IP}^t = VA^t - P_{IP}^tQ_{IP}^t$ is Comprehensive Value Added from (10), and $[r^t - i^t + (1+i^t)\delta^t]P_K^{t-1} = U^t$ is the user cost of capital from (6):[18]

[17] This is the case where i^t is negative.

[18] Balk (2010; S239-S247) introduced many more rows to Table 1 by decomposing user cost into four separate components and then shifting these components from the input column to the output column. We note that his decomposition of user cost into separate components is slightly different from our decomposition. Balk correctly includes property taxes in user cost so this adds the term $\tau P_K^{t-1}Q_K^{t-1}$ to the income column in Table 1. Thus we regard property taxes paid by the Production Unit as a contribution to all of the income concepts defined in Table 1. Our r^t is a gross rate of return that includes income taxes paid by the Production Unit so income taxes also contribute to all forms of income defined in Table 1.

Table 1: Alternative Output and Corresponding Income Concepts

Output Concepts	Income Concepts
$GDO^t = CVA^t + P_K^t Q_I^t$	$GDI^t = P_L^t Q_L^t + [r^t - i^t + (1+i^t)\delta^t]P_K^{t-1}Q_K^{t-1} + \Pi^t$
$NDO^t = GDO^t - (1+i^t)\delta^t P_K^{t-1}Q_K^{t-1}$	$NDI^t = P_L^t Q_L^t + [r^t - i^t]P_K^{t-1}Q_K^{t-1} + \Pi^t$
$CNDO^t = NDO^t + i^t P_K^{t-1}Q_K^{t-1}$	$CNDI^t = P_L^t Q_L^t + r^t P_K^{t-1}Q_K^{t-1} + \Pi^t$

Following Balk (2010), one can define (one plus) *Productivity Growth* (or Total Factor Productivity Growth) of the Production Unit in time period t relative to a base period 0, $Prod^t$, as the Fisher (1922) quantity index of (net) outputs relating period t to period 0 divided by the corresponding Fisher quantity index of inputs.[19] For each row in Table 1, there is a different productivity measure. For the Gross Output concept, the period t output prices are P_Y^t, P_Z^t, P_{IP}^t and P_K^t and the corresponding period t quantities are Q_Y^t, Q_Z^t, $-Q_{IP}^t$ and Q_I^t. The corresponding period t input prices are P_L^t and $U^t = [r^t - i^t + (1+i^t)\delta^t]P_K^{t-1}$and the period t input quantities are Q_L^t and Q_K^{t-1}. For the Comprehensive Net Income concept, the period t output prices are P_Y^t, P_Z^t, P_{IP}^t, P_K^t and $[i^t - (1+i^t)\delta^t]P_K^{t-1}$ and the corresponding period t quantities are Q_Y^t, Q_Z^t, $-Q_{IP}^t$, Q_K^t and Q_K^{t-1}. The period t input prices are P_L^t and $r^t P_K^{t-1}$ and the corresponding period t input quantities are Q_L^t and Q_K^{t-1}. Note that pure profits Π^t do not appear in either the output or input index numbers in this Balkian framework.

Choose a row in Table 1 and denote the period t output price and quantity vectors by p^t and y^t. Denote the period t input price and quantity vectors by w^t and x^t. Denote the Fisher output and input price and quantity indexes for period t relative to period 0 by $P_F(p^0, p^t, y^0, y^1) = [p^t \cdot y^0\, p^t \cdot y^t / p^0 \cdot y^0 p^0 y^t]^{1/2}$ and $Q_F(p^0, p^t, y^0, y^t) = [p^0 \cdot y^t p^t \cdot$

[19] The idea of defining TFP growth as an output index divided by an input index goes back to Jorgenson and Griliches (1967). Balk probably chose the Fisher index as his functional form for price and quantity indexes because of its superior axiomatic properties; see Diewert (1992). Balk's accounting approach to productivity measurement draws on Diewert (1990) and Diewert and Nakamura (2003) but is more general since Balk allows profits to be nonzero.

$y^t/p^0 \cdot y^0 p^t \cdot y^0]^{1/2}$ (for outputs) and $P_F(w^0,w^t,x^0,x^t) = [w^t \cdot x^0 w^t \cdot x^t/w^0 \cdot x^0 w^0 x^t]^{1/2}$ and $Q_F(w^0,w^t,x^0,x^t) = [w^0 \cdot x^t w^t \cdot x^t/w^0 \cdot x^0 w^t \cdot x^0]^{1/2}$ (for inputs). Thus $Prod^t = Q_F(p^0,p^t,y^0,y^t)/Q_F(w^0,w^1,x^0,x^1)$ and Balk's (2010: S233) *growth accounting decomposition* into explanatory factors for the output/income concept defined by $p^t \cdot y^t$ is the following identity:

$$\frac{p^t \cdot y^t}{p^0 \cdot y^0} = Prod^t \times \frac{Q_F(w^0,w^t,x^0,x^t)}{P_F(p^0,p^t,y^0,y^t)}. \qquad (16)$$

Thus (one plus) nominal output/income growth is equal to Productivity growth times (one plus) input quantity growth divided by (one plus) output price growth.[20]

4. Discussion of Alternative Approaches to Firm Accounting

We will attempt to clarify some of our definitions and provide additional discussion about our accounting framework.[21]

4.1 Observed Prices and Quantities versus Imputed Prices and Quantities

It is useful to distinguish a production unit's *actual* (observable) period t revenues and costs from *imputed* costs and revenues. Period t prices and quantities that are in principle observable are revenues $P_Y^t Q_Y^t$, intermediate input costs $P_Z^t Q_Z^t$, purchased investments $P_{IP}^t Q_{IP}^t$ and labour costs $P_L^t Q_L^t$.[22] Imputed variables in our accounting framework are the

[20] Using the Törnqvist index number formula in place of the Fisher formula, Kohli (1990) was able to obtain a growth accounting decomposition that was more detailed, i.e., the output price index and the input quantity index were decomposed into individual price and quantity explanatory factors; see also Diewert and Morrison (1986). These authors assumed that profits were equal to zero.

[21] This section was added in response to the comments of the referees on an earlier draft.

[22] As was indicated in section 2, the quantities are total amounts purchased or sold during period t and the corresponding prices are unit value prices. The use of unit value prices to aggregate over transactions made during the accounting period was recommended by early index number theorists; see Walsh (1901; 96) and Fisher (1922; 318). If units of the capital stock are sold during the accounting period, then Q_{IP}^t becomes net asset purchases and could become negative if asset sales are bigger than asset purchases.

prices and quantities of the capital stock at the beginning and end of the period, P_K^{t-1}, P_K^t, Q_K^{t-1} and Q_K^t,the price and quantity of own account investment, P_{II}^t and Q_{II}^t, the period t cost of financial capital (the reference interest rate) r^t and the period t rate of geometric depreciation δ^t. Accounting theorists and practitioners have long stressed the importance of using actual data[23] and the difficulties associated with the use of imputed data.[24] However, in order to evaluate firm performance over a given (short) time period, it is necessary to value capital stocks at the beginning and end of the accounting period. This valuation exercise involves a model of depreciation of the capital stocks and a model for pricing depreciated capital stocks. Thus imperfect imputations are required in order to evaluate firm performance over the accounting period. These valuation problems are caused by the durability of capital inputs in the production process.[25] In the following paragraph, we will define various aggregates that are based on observable data.

The four categories of observable revenues and costs can be combined in various ways in order to define the following *observable aggregates*:

$$\begin{aligned}
VA^t &\equiv P_Y^t\ Q_Y^t - P_Z^t\ Q_Z^t: && \text{Value Added;} \qquad (17)\\
CVA^t &\equiv VA^t - P_{IP}^t\ Q_{IP}^t: && \text{Comprehensive Value Added;}\\
CF^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_L^t Q_L^t: && \text{Cash Flow;}\\
CCF^t &\equiv CF^t - P_{IP}^t Q_{IP}^t: && \text{Comprehensive Cash Flow}
\end{aligned}$$

The above definitions for period t Value Added and Cash Flow are reasonably well established in the economics and accounting literature.[26] Our definitions for CVA^t and

[23] Accounting theorists have stressed the importance of using transactions data which are objective, reliable and reproducible; see Daines (1929; 99-101) and Ijiri (1979) on objectivity, Canning (1929; 321) on reliability and Davidson, Stickney and Weil (1976; 225) on reproducibility.

[24] See Daines (1929; 98) and Ijiri (1979; 66).

[25] "The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is the fundamental problem of accounting." W. Erwin Diewert (2005a; 480).

[26] In the accounting literature, our Cash Flow is roughly equivalent to Cash Flow from Operations. Our measure of Comprehensive Cash Flow includes (net) purchases of the investment good. Our comprehensive measure is not a truly comprehensive measure because it excludes transactions in financial markets that determine the production unit's cost of capital, r^t. For models that integrate financial transactions into the Neo-Austrian model, see Diewert (2014) and Diewert, Fixler and Zieschang (2016).

CCF^t simply subtract market expenditures on the investment good,[27] $P_{IP}^t Q_{IP}^t$, from VA^t and CF^t, respectively. Using the above definition of Value Added, definition (1) for period t *pure profits* Π^t of the production unit can be written as follows:

$$\Pi^t = VA^t - P_L^t Q_L^t - P_{IP}^t Q_{IP}^t + P_K^t Q_K^t - (1+r^t) P_K^{t-1} Q_K^{t-1} . \qquad (18)$$

4.2 Can Neo-Austrian Profit be Written as a Flow?

A referee pointed out that our definition (1) of pure profit involved a mixture of stock and flow variables and one can ask whether pure profits can be rewritten purely in terms of flow variables. Using the geometric model of depreciation, we showed that pure profits Π^t defined by (1) are equal to the expression on the right hand side of (5). Using definition (17) of period t cash flow CF^t, (5) can be rewritten as follows:

$$\Pi^t = CF^t - U^t Q_K^{t-1} + P_K^t Q_I^t - P_{IP}^t Q_{IP}^t . \qquad (19)$$

The user cost price of the beginning of the period capital stock, U^t, was defined by (6). $U^t Q_K^{t-1}$ is conceptually equal to the cost of renting the initial capital stock and hence is a flow variable. If we use (6) to decompose the user cost into its components, then we have the following decomposition:

$$\begin{aligned} U^t Q_K^{t-1} &= r^t P_K^{t-1} Q_K^{t-1} - i^t P_K^{t-1} Q_K^{t-1} + (1+i^t)\delta^t P_K^{t-1} Q_K^{t-1} \qquad (20) \\ &= \text{interest cost} - \text{ revaluation} + \text{ depreciation.} \end{aligned}$$

Thus the various components of the cost of using the initial capital stock can also be decomposed into flows. We also need to rewrite the final two terms on the right hand side of (19) in terms of flows that make sense. Replace total period t investment Q_I^t by the sum

[27] If the production unit sells part of its beginning of the period capital stock during period t, then Q_{IP}^t is interpreted as net (market) purchases of the investment good and if period t asset sales are bigger than asset purchases, then Q_{IP}^t becomes negative.

of own account investment $Q_{II}^t$and purchased investment Q_{IP}^t. This leads to the following equations:

$$
\begin{aligned}
P_K^t Q_I^t - P_{IP}^t Q_{IP}^t &= P_K^t (Q_{II}^t + Q_{IP}^t) - P_{IP}^t Q_{IP}^t \\
&= (P_K^t - P_{IP}^t) Q_{IP}^t + P_K^t Q_{II}^t.
\end{aligned}
\qquad (21)
$$

The term $P_K^t Q_{II}^t$ is the imputed value of own account investment valued at the end of period price for a unit of the capital stock which is P_K^t. This term is a flow. The term $(P_K^t - P_{IP}^t) Q_{IP}^t$ is a *revaluation term* for purchased investment and hence is also a flow variable. This term will contribute to period t profits if the end of period price of an investment good, P_K^t, is greater than the within the period purchase price for the investment good,P_{IP}^t. The terms on the right hand side of (21) are flows so it is possible to interpret our measure of pure profits in terms of period t flows.

There is no explicit revaluation term for own account investment because there is no explicit purchase price for this type of investment. The cost of own account investment is included in intermediate input, labour and capital services that were used to produce Q_{II}^t. If these costs could be separated from the overall costs $P_Z^t Q_Z^t$, $P_L^t Q_L^t$ and $U^t Q_K^{t-1}$, then these separated costs could be cumulated and divided by Q_{II}^t to give us an estimated (or imputed) price P_{II}^t. One could then use the new costs of intermediates, labour and capital services along with a new cost category, $P_{II}^t Q_{II}^t$, and the decomposition (21) would be replaced by the symmetric decomposition $P_K^t Q_I^t - P_{IP}^t Q_{IP}^t = (P_K^t - P_{IP}^t) Q_{IP}^t + (P_K^t - P_{II}^t) Q_{II}^t$.[28]

Note that that the flow decomposition defined by (21) can be applied to our definition of Gross Domestic Output, GDO^t, defined by (9). Using (9), (17) and (21), we have:

$$
\begin{aligned}
GDO^t &= VA^t + P_K^t Q_I^t - P_{IP}^t Q_{IP}^t \\
&= VA^t + P_K^t Q_{II}^t + (P_K^t - P_{IP}^t) Q_{IP}^t.
\end{aligned}
\qquad (22)
$$

[28] If the investment good is being produced by the production unit, then sales of the good would appear as a revenue item. Thus own account production is interpreted as production of the investment good for use by the production unit for its own use in the following period.

Thus Neo-Austrian Gross Domestic Output is equal to traditional Value Added VA^t plus Own Account Investment valued at the end of period investment price $P_K^t Q_{II}^t$ plus Revaluation Gains on purchased investment $(P_K^t - P_{IP}^t)Q_{IP}^t$. Thus if there is no own account investment and the end of period price of a unit of the capital stock is P_K^t is set equal to the average period price of capital stock purchases P_{IP}^t, then the last two terms on the right hand side of (22) vanish and our $GDO^t$is equal to traditional value added VA^t.

4.3 Should Asset Price Change be Added to Net Output?

Schreyer (2009; 50-51) has an extensive discussion on alternative net income concepts and he updates the Pigou (1941)-Hayek (1941) controversy on whether income concepts should hold constant the physical capital stock (the Pigou position) or the real financial capital stock (the Hayek position). In our accounting framework, this controversy boils down to a choice between Net Domestic Output or Comprehensive Net Domestic Output.

It has long been recognized that measures of Gross Domestic Output overstate the value to society of production because depreciation of the beginning of the period capital stock is not deducted from measures of gross output.[29] Thus from a theoretical point of view, deducting depreciation from the measure of gross output has not been controversial. However, adding capital gains (or losses) to a measure of net output has been resisted by national income accountants. Schreyer explained why the current System of National Accounts does not add the value of (net) capital gains on the initial capital stock (the term $i^t P_K^{t-1} Q_K^{t-1}$ to the value of net output:

"The present Manual uses a notion of depreciation that does not encompass the changes in relative prices of assets. There are several reasons for this.

[29] For example, see Samuelson (1961), Schreyer (2009; 43) and Balk (2010; S244) for discussions of this issue. Before the use of memory chips became widespread, measures of gross and net output tended to move in a proportional manner, so growth rates of gross and net domestic product were similar. However, Spant (2003) showed empirically that this similarity in growth rates no longer holds.

● The first reason is that it keeps the supply side and production perspective of the economy separate from the demand and consumer side. A measure of depreciation that captures the discounted value of capital used up in production and the investment needed to keep the productive capacity of the economy intact fits into a supply-side perspective. A consumer or demand side perspective can easily be added by considering wealth effects arising with the ownership of productive assets but it seems better to keep these effects separate rather than lumping them together in the first place.

● The second reason is that present practice in OECD countries' national accounts corresponds to a notion of depreciation that excludes wealth effects. Also, if one wanted to bring real wealth effects into measures of depreciation, there is a question whether such effects should be integrated asymmetrically (capturing only expected real holding losses) or symmetrically (allowing also for real holding gains). However, we reiterate that different analytical questions may give rise to different treatment of relative price changes for capital goods. In particular, for the analysis of wealth effects and associated welfare considerations, it is meaningful to account for real price changes. Net income would then decline in the presence of expected holding losses and rise in the presence of expected holding gains." Schreyer (2009; 51).

There is a third reason to exclude holding gains from a measure of net output: asset price inflation, i^t, can be very large and positive (and negative) and thus the addition of the term $i^t P_K^{t-1} Q_K^{t-1}$ to the measure of net output can lead to an income measure that is extremely volatile. Our suggested solution to this volatility problem is to replace actual *ex post* asset price inflation rates by smoothed asset inflation rates.[30] Thus computing a nonvolatile measure of comprehensive net output requires two major imputation models: a model of depreciation and a model for smoothing asset prices.

[30] This volatility problem shows up in the user cost of land which can easily become negative if ex post asset inflation rates are used as the i^t. The use of smoothed asset price inflation rates in the user cost formula will tend to eliminate negative user costs; see Diewert and Fox (2018).

It is unfortunate that a useful measure of comprehensive net income generated by a production unit requires so many imputations, but we believe it is important for statistical offices to provide a measure of comprehensive net income due to the increasing importance of land as a factor of production. In many countries, the value of land is comparable to the value of reproducible capital and land prices have been increasing over past decades. Thus capital gains on land holdings have become an important source of income which is not being measured by many countries.

The above discussion can be summarized as follows:

- Gross Domestic Output is a useful measure of output because it can be produced by National Statistical Offices without making a lot of imputations. Thus it can be regarded as a more reliable measure of output.
- Net Domestic Output is also a useful measure of output that better reflects sustainable output. It requires some imputations in order to determine depreciation and smoothed asset inflation rates.
- Comprehensive Net Domestic Output is a useful measure of the income generated by the production sector. In order to avoid huge fluctuations in this measure, smoothed asset inflation rates should be used. This measure of output requires three sets of imputations: one for determining depreciation (and capital stocks), one for determining beginning of the period asset prices and one for determining smoothed asset inflation rates.

Our Neo-Austrian approach to the valuation of investment is consistent with current value accounting theory since our methodology follows exactly the approach of Edwards and Bell (1961) who are respected accounting theorists. Moreover, if we deflate our measure of Comprehensive Net Domestic Income, $CNDI^t = P_L^t Q_L^t + r^t P_K^{t-1} Q_K^{t-1} + \Pi^t$, by the country's Consumer Price Index for the end of period t, P_{CPI}^t, then we obtain a measure of real income generated by the Production Unit that was recommended by the accountant Sterling (1975; 50). Thus our approach to firm accounting unifies national income accounting theory with business firm accounting.

In the following subsection, we specialize the Neo-Austrian approach to accounting to the problems associated with the treatment of land.[31]

4.4 The Treatment of Land

The algebra in section 4.2 can be applied to a Production Unit that uses land services as an input. For simplicity, assume that land is the single asset used in production. Thus Q_K^{t-1} is the amount of land available to the Production Unit (PU) at the beginning of period t and its (imputed) price is P_K^{t-1}. The PU may purchase additional units of land during period t Q_{IP}^t at the price P_{IP}^t. It may be the case that the PU converts undeveloped land into higher quality land so own account production of land, Q_{II}^t could be positive. For simplicity, we will assume that there is no own account investment in land development. Thus period t investment in land Q_I^t is equal to Q_{II}^t and the corresponding investment price P_I^t is equal to P_{IP}^t. Thus gross (and net) investment in land during period t is equal to the difference between the end of period and the beginning of period quantities of land:

$$Q_I^t = Q_K^t - Q_K^{t-1} \,. \qquad (23)$$

Equation (23) is consistent with the geometric model of depreciation if we set the period t depreciation rate δ^t equal to zero. If $Q_I^t > 0$, then P_I^t is the purchase price for newly acquired land; if $Q_I^t < 0$, then P_I^t is the observed selling price for sold land. With these assumptions, pure profits for the PU are defined as follows:

$$\begin{aligned} \Pi^t &= CF^t + P_I^t\, Q_I^t + P_K^t Q_K^t - (1+r^t)P_K^{t-1}Q_K^{t-1} && (24) \\ &= CF^t + P_I^t(Q_K^t - Q_K^{t-1}) + \; P_K^t Q_K^t - (1+r^t)P_K^{t-1}Q_K^{t-1} && \text{using (23)} \\ &= CF^t + (P_K^t - P_I^t)Q_I^t - U^t Q_K^{t-1}. \end{aligned}$$

[31] For a specialization of the Neo-Austrian approach to the treatment of inventory change, see Diewert (2005b), and for a specialization to the resource depletion context, see Diewert and Fox (2016).

where the *user cost of capital* is defined as $U^t = (r^t - i^t)P_K^{t-1}$when the depreciation rate $\delta^t = 0$. Thus pure profits are equal to cash flow less the user cost of land plus the term $(P_K^t - P_I^t)Q_I^t$ which is equal to the end of period capital gains or losses on the (net) purchases of land made during the accounting period. Typically, this capital gains term will be small.

Since the depreciation rate for land is zero, our measures of gross and net domestic output, GDO^t and NDO^t, will be equal. Thus for land investments, Table 1 in section 3 becomes Table 2.

Table 2: Gross Output and Comprehensive Net Output for Land Investments

Output Concepts	Income Concepts
$GDO^t = VA^t + (P_K^t - P_I^t)Q_I^t$	$GDI^t = P_L^t Q_L^t + [r^t - i^t]P_K^{t-1}Q_K^{t-1} + \Pi^t$
$CNDO^t = GDO^t + i^t P_K^{t-1}Q_K^{t-1}$	$CNDI^t = P_L^t Q_L^t + r^t P_K^{t-1}Q_K^{t-1} + \Pi^t$

A number of points of interest emerge from a study of Table 2:

- The asset inflation rate for land, i^t, can exceed the reference cost of capital, r^t, and so the user cost of capital in this case, $[r^t - i^t]P_K^{t-1}$, becomes a *user benefit*.
- Our GDO concept differs from national accounts GDP by adding the asset revaluation term $(P_K^t - P_I^t)Q_I^t$. As was mentioned above, for an individual production unit, this revaluation term will usually be small for an individual firm or sector. However, when we aggregate over production units in the national economy, the $P_I^t Q_I^t$ terms will sum to zero, so effectively, we are adding the term $P_K^t Q_I^t$ term to value added to obtain our Neo-Austrian measure of gross output. In many economies, agricultural land (which has a low price) is converted into commercial, industrial and residential land (which tends to have a much higher price). Thus in aggregate, adding the terms $P_K^t Q_I^t$ for the different types of land to value added will tend to give a significant boost to our measure of gross output.

- Our measure of Comprehensive Net Output adds the capital gains (or losses if i^t is negative) on the value of land over the accounting period, $i^t P_K^{t-1} Q_K^{t-1}$, to gross output. This term can be very large. Thus it is important to include land in the list of productive assets when constructing a measure of income generated by the production sector of an economy.

It should be noted that our preferred measure of the income generated by a production unit is only loosely related to the aggregate income of the residents of a country; i.e., some fraction of the domestic capital stock will be owned by nonresidents and thus some of the returns generated by the production unit will flow to nonresident owners. There are many additional measurement problems that we have not addressed in this paper.[32]

5. Economy Wide Measures of Output, Input and Income

In this section, we extend the analysis to many types of capital and we also aggregate over production units. Suppose there are F production units in the economy, N types of capital, J classes of outputs (including outputs of capital goods) and M classes of intermediate inputs (excluding capital good purchases).[33] The counterparts to definitions (1)-(6) will be explained below.

Define the *period t pure profits of production unit* f, Π_f^t, as follows, for $f = 1, \ldots, F$:

[32] The reader is directed to the work of Stiglitz, Sen and Fitoussi (2009), Schreyer (2009) and Balk (2010) (2011) for extended discussions of the many important measurement problems associated with measuring gross and net output for the production accounts.

[33] To keep our notation as simple as possible, we have only one type of labour in the economy. The algebra can readily be generalized to many types of labour.

$$\Pi_f^t \equiv \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t - \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t \tag{25}$$

$$- P_{Lf}^t\, Q_{Lf}^t + \sum_{n=1}^{N} P_{Kfn}^t Q_{Kfn}^t - (1+r^t) \sum_{n=1}^{N} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}$$

$$= VA_f^t - P_{Lf}^t\, Q_{Lf}^t + \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t + \sum_{n=1}^{N} P_{Kfn}^t Q_{Kfn}^t$$

$$- (1+r^t) \sum_{n=1}^{N} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}$$

The *Value Added* for production unit f, VA_f^t, is defined as follows, for $f = 1, \ldots, F$:

$$VA_f^t \equiv \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t . \tag{26}$$

The various price and quantity variables appearing on the right hand side of definitions (25) and (26) are defined as follows:

P_{Yfj}^t ≡ (unit value) price of output j sold by production unit f during period t;

Q_{Yfj}^t ≡ total quantity of output j produced by unit f during period t;

P_{Zfm}^t ≡ (unit value) price of intermediate input m purchased by unit f during period t;

Q_{Zfm}^t ≡ total quantity purchased of intermediate input m purchased by unit f during period t;

P_{IPfn}^t ≡ (unit value) price of one unit of investment good n *purchased* by unit f during period t;

Q_{IPfn}^t ≡ total number of units of the investment good n *purchased* by unit f during period t;

P_{Lf}^t ≡ wage rate for one hour of labour used by unit f during period t;

Q_{Lf}^t ≡ total hours worked for unit f in period t;

P_{Kfn}^t ≡ price of a unit of capital stock n held by unit f at the end of period t;

Q_{Kfn}^{t} ≡ quantity of capital stock n held by unit f at the end of period t;

P_{Kfn}^{t-1} ≡ price of a unit of the capital stock n held by unit f at the beginning of period t;

Q_{Kfn}^{t-1} ≡ quantity of capital stock n held by unit f at the beginning of period t;

r^{t} ≡ period t cost of capital for all production units.

The assumption that the cost of capital r^t is constant across all production units is only a very rough approximation to reality. We make this assumption because at a later stage of our analysis, we adapt our algebra to the problem of determining an economy wide *ex post* rate return on capital.

We have defined P_{IPfn}^{t} and Q_{IPfn}^{t} as the period t price and quantity of purchases of investment good n by production unit f. However, each production unit may also produce units of the investment good internally for its own use. Thus define $Q_{IIfn}^{t} \geq 0$ as the amount of internally produced investment good n by unit f and P_{IIfn}^{t} as the corresponding imputed price for a unit of this internally produced investment. Define period t *total investment* in the n^{th} capital stock by production unit f as the sum of purchased investment, Q_{IPfn}^{t}, plus internally produced investment, Q_{IIfn}^{t}:

$$Q_{Ifn}^{t} \equiv Q_{IPfn}^{t} + Q_{IIfn}^{t}; \quad f = 1, \ldots, F; n = 1, \ldots, N. \qquad (27)$$

As in the previous section, we assume that *geometric depreciation* applies to each capital stock. Thus we assume that the following relationships between beginning and end of period capital stocks and total investment hold:

$$Q_{Kfn}^{t} = (1 - \delta_n^t) Q_{Kfn}^{t-1} + Q_{Ifn}^{t}; \quad f = 1, \ldots, F; n = 1, \ldots, N. \qquad (28)$$

Note that the period t geometric depreciation rate for the n^{th} type of capital, $\delta_n^{t\,t}$, depends on t and n but not on f. Using these assumptions, it can be shown that we can obtain the following expression for the pure profits of Production Unit f for $f = 1,\ldots,F$:[34]

$$\Pi_f^t = \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t - P_{Lf}^t Q_{Lf}^t - \sum_{n=1}^{N} U_{fn}^t Q_{Kfn}^{t-1} + \sum_{n=1}^{N} P_{Kfn}^t Q_{Ifn}^t - \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t \quad (29)$$

where the *user cost of capital stock component n for unit f* is defined as $U_{fn}^t \equiv \left[r^t - i_{fn}^t + \left(1 + i_{fn}^t\right)\delta_n^t\right] P_{Kfn}^t$ and the *capital stock asset inflation rates* i_{fn}^t are defined by $(1 + i_{fn}^t) \equiv P_{Kfn}^t / P_{Kfn}^{t-1}$ for $f = 1,\ldots,F$ and $n = 1,\ldots,N$. In what follows, we make the simplifying assumption that for each asset n, the inflation rate for each production unit is constant, i.e., we assume that $i_{nf}^t = i_n^t$ for $f = 1,\ldots,F$ and $n = 1,\ldots,N$.

For each class of the six quantity variables on the right hand side of (29), define the corresponding *national aggregate* by summing over production units. Thus $Q_{Yj}^t \equiv \sum_{f=1}^{F} Q_{Yfj}^t$ for $j = 1,\ldots,J$; $Q_{Zm}^t \equiv \sum_{f=1}^{F} Q_{Zfm}^t$ for $m = 1,\ldots,M$; $Q_L^t \equiv \sum_{f=1}^{F} Q_{Lf}^t$; $Q_{Kn}^{t-1} \equiv \sum_{f=1}^{F} Q_{Kfn}^{t-1}$ for $n = 1,\ldots,N$; $Q_{In}^t \equiv \sum_{f=1}^{F} Q_{Ifn}^t$ for $n = 1,\ldots,N$ and $Q_{IPn}^t \equiv \sum_{f=1}^{F} Q_{IPfn}^t$ for $n = 1,\ldots,N$.

Define the corresponding *national unit value prices* as follows: $P_{Yj}^t \equiv [\sum_{f=1}^{F} P_{Yfj}^t Q_{Yfj}^t]/Q_{Yj}^t$ for $j = 1,\ldots,J$; $P_{Zm}^t \equiv [\sum_{f=1}^{F} P_{Zfm}^t Q_{Zfm}^t]/Q_{Zm}^t$ for $m = 1,\ldots,M$; $P_L^t \equiv [\sum_{f=1}^{F} P_{Lf}^t Q_{Lf}^t]/Q_L^t$; $U_n^t \equiv [\sum_{f=1}^{F} U_{fn}^t Q_{Kfn}^{t-1}]/Q_{Kn}^{t-1}$ for $n = 1,\ldots,N$; $P_{In}^t \equiv [\sum_{f=1}^{F} P_{Kfn}^t Q_{Ifn}^t]/Q_{In}^t$ for $n = 1,\ldots,N$;[35] and $P_{IPn}^t \equiv [\sum_{f=1}^{F} P_{IPfn}^t Q_{IPfn}^t]/Q_{IPn}^t$ for $n = 1,\ldots,N$.

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[34] Equations (29) are production unit counterparts to equations (19) above.

[35] Note that the unit value price for total period t investment in asset n, P_{In}^t is equal to $\sum_{f=1}^{F} P_{Kfn}^t Q_{Ifn}^t$ divided by total investment in asset n, $Q_{In}^t \equiv \sum_{f=1}^{F} Q_{Ifn}^t$. Using the Hicks, Edwards and Bell accounting framework leads to total investment being valued at end of period prices for the various assets.

Finally, define *national value added*, VA^t, by summing value added over the production units:

$$VA^t \equiv \sum_{f=1}^{F}\left[\sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t\right]. \tag{30}$$

National pure profits, Π^t, are obtained by summing the production unit profits Π_f^t defined by (29). Using the above definitions, we obtain the following decomposition of national pure profits into national explanatory aggregates:

$$\begin{aligned}\Pi^t &\equiv \sum_{f=1}^{F} \Pi_f^t \\ &= VA^t - P_L^t Q_L^t - \sum_{n=1}^{N} U_n^t Q_{Kn}^{t-1} + \sum_{n=1}^{N} P_{In}^t Q_{In}^t - \sum_{n=1}^{N} P_{IPn}^t Q_{IPn}^t .\end{aligned} \tag{31}$$

The definition and decomposition of profits given by (31) is the macroeconomic counterpart to the microeconomic decomposition of profits given by (19). Using (31), period t Neo-Austrian National Gross Domestic Output is defined by (32) and the companion Gross Domestic Income is defined by (33):

$$GDO^t \equiv VA^t + \sum_{n=1}^{N} P_{In}^t Q_{In}^t - \sum_{n=1}^{N} P_{IPn}^t Q_{IPn}^t; \tag{32}$$

$$GDI^t \equiv P_L^t Q_L^t + \textstyle\sum_{n=1}^{N} U_n^t Q_{Kn}^{t-1} + \Pi^t. \tag{33}$$

In order to define Neo-Austrian National Net Domestic Output, we need to decompose user costs into various components. Define the *national beginning of period t stock of asset* n as $Q_{Kn}^{t-1} \equiv \sum_{f=1}^{F} Q_{Kfn}^{t-1}$ for $n = 1, \ldots, N$. Define the corresponding unit value prices as $P_{Kn}^{t-1} \equiv \sum_{f=1}^{F} P_{Kfn}^{t-1} Q_{Kfn}^{t-1} / Q_{Kn}^t$ for $n = 1, \ldots, N$. Under our assumptions, we can rewrite $\sum_{n=1}^{N} U_n^t Q_{Kn}^{t-1}$ as follows:

$$\begin{aligned}\sum_{n=1}^{N} U_n^t Q_{Kn}^{t-1} &= \sum_{n=1}^{N}\sum_{f=1}^{F} U_{fn}^t Q_{Kfn}^{t-1} \\ &= \sum_{n=1}^{N}\sum_{f=1}^{F}[r^t - i_n^t + (1+i_n^t)\delta_n^t]P_{Kfn}^{t-1}Q_{Kfn}^{t-1} \\ &= \sum_{n=1}^{N}[r^t - i_n^t + (1+i_n^t)\delta_n^t]\sum_{f=1}^{F} P_{Kfn}^{t-1}Q_{Kfn}^{t-1} \\ &= \sum_{n=1}^{N}[r^t - i_n^t + (1+i_n^t)\delta_n^t]P_{Kn}^{t-1}Q_{Kn}^{t-1} \\ &= \sum_{n=1}^{N}[r^t - i_n^t]P_{Kn}^{t-1}Q_{Kn}^{t-1} + \sum_{n=1}^{N}(1+i_n^t)\delta_n^t P_{Kn}^{t-1}Q_{Kn}^{t-1}\ .\end{aligned} \tag{34}$$

To obtain *National Net Domestic Output*, NDO^t, use (34) and simply shift the depreciation terms, $\sum_{n=1}^{N}(1+i_n^t)\delta_n^t P_{Kn}^{t-1}Q_{Kn}^{t-1}$, from the input side of the accounts to the output side. Thus we have the following definitions for NDO^t and the companion input or income concept NDI^t:

$$NDO^t \equiv GDO^t - \sum_{n=1}^{N}(1+i_n^t)\delta_n^t P_{Kn}^{t-1}Q_{Kn}^{t-1}\,; \tag{35}$$

$$NDI^t \equiv P_L^t Q_L^t + \sum_{n=1}^{N}[r^t - i_n^t]P_{Kn}^{t-1}Q_{Kn}^{t-1} + \Pi^t. \tag{36}$$

Finally, to obtain *Comprehensive Net Domestic Output*, $CNDO^t$, shift (minus) capital gains on the value of the initial national capital stock, $-\sum_{n=1}^{N} i_n^t P_{Kn}^{t-1}Q_{Kn}^{t-1}$, from the input side of the accounts to the output side. Thus we have the following definitions for $CNDO^t$ and the companion input or income concept $CNDI^t$:

$$CNDO^t \equiv NDO^t + \sum_{n=1}^{N} i_n^t P_{Kn}^{t-1} Q_{Kn}^{t-1}\,; \qquad (37)$$

$$CNDI^t \equiv P_L^t Q_L^t + \sum_{n=1}^{N} r^t P_{Kn}^{t-1} Q_{Kn}^{t-1} + \Pi^t. \qquad (38)$$

The interpretation of the various macroeconomic concepts follows along the same lines as our discussions of the microeconomic concepts. However, there is a reduction in data requirements when we move to the national level from the individual firm or sectoral level: intermediate input transactions cancel out when we do the aggregation. If we focus on production unit deliveries to final demand, we do not need to collect data on intermediate input transactions.

The flow outputs of a PU are delivered to: (i) other domestic Production Units who use the delivered outputs as intermediate inputs or as additions to their capital stocks; (ii) domestic households; (iii) the general government sector or (iv) the export sector. The flow inputs used by a PU come from either domestic producers or imports.

Thus it can be seen that aggregate value added is equal to the value of household expenditures on consumer goods and services (valued at producer prices)[36] plus the value of government (net) purchases of goods and services from the private production sector plus the value of exports (before export taxes) less the value of imports (after import taxes) plus the aggregate value of purchased investments. Suppose we have period t price and quantity indexes for these four components of final demand, say P_C^t, P_G^t, P_x^t and P_M^t for prices and Q_C^t, Q_G^t, Q_x^t and Q_M^t for quantities.[37] Then it can be seen that the following equality holds:

[36] Jorgenson and Griliches (1972) noted the importance of using prices that producers face in productivity studies. If an output of a domestic producer is taxed, then the producer only gets the before tax price to add to revenue; if an imported good or service is taxed, then the producer faces the after tax price and the after tax value of the input should be added to producer cost.

[37] However, there is a problem with taxed intermediate inputs that are produced domestically and purchased by a domestic final demander. The tax revenue raised by this internal commodity tax does not cancel out as we aggregate over units. For more on the treatment of taxes in the production accounts, see Diewert (2006).

$$VA^t = P_C^t Q_C^t + P_G^t Q_G^t + P_X^t Q_X^t - P_M^t Q_M^t + \sum_{n=1}^{N} P_{IPn}^t Q_{IPn}^t. \tag{39}$$

Now replace VA^t in (32), which defined Gross Domestic Output GDO^t, and we obtain the following expression:

$$GDO^t \equiv P_C^t Q_C^t + P_G^t Q_G^t + P_X^t Q_X^t - P_M^t Q_M^t + \sum_{n=1}^{N} P_{In}^t Q_{In}^t. \tag{40}$$

Neo-Austrian GDO^t is essentially equal to standard expenditure side GDP at producer prices except that gross investment is valued at end of period prices instead of at the average prices of investment transactions during period t. Thus our economy wide various output and input measures defined above can be computed using standard macroeconomic data for an economy.

6. Conclusion

We have systematically introduced alternative output, input and income concepts, for both individual production units (such as firms) and at aggregate levels. The differences in definitions have their roots in an Austrian model of production (Böhm-Bawerk 1891) and the debate between Pigou (1941) and Hayek (1941) on the maintenance of physical versus financial capital.

This paper contributes to the literature by making clear the definitions and their relationships, highlighting how each provides a different perspective. For example, each definition of output (at both individual production unit and aggregate levels) provides a different perspective of production. Use of price deflated versions of these output concepts in productivity studies will typically lead to different perspectives on productivity performance. Similarly for the primary output/income concepts.

Researchers using firm level data can use the results on individual production units from section 3 to provide an enhanced view of sources of firm performance. More importantly for economic management, the aggregate measures presented in section 5 could be calculated by national statistical offices, providing macroeconomists and productivity researchers with additional information that can be used to better inform policy.

We are not advocating the abandonment of GDP; it is a useful measure that serves many purposes and has the advantage of requiring a minimal number of imputations. But it would be useful to have a supplementary input measure that better approximated the income generated by domestic producers.

Finally, we note that our accounting approach is based on a branch of commercial accounting theory and thus our approach reconciles commercial accounting with national income accounting.

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