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Tomáš Želinský

(Technical University of Košice and Durham University)

Martina Mysíková

(Czech Academy of Sciences)

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Sensitivity of the income poverty headcount ratio to equivalence scale parameters[1]

Tomáš Želinský
Faculty of Economics, Technical University of Košice,
Němcovej 32, 04001 Košice, Slovakia

Durham University, School of Education,
Leazes Road, Durham DH1 1TA, UK
Email: tomas.zelinsky@tuke.sk

Martina Mysíková
Institute of Sociology, Czech Academy of Sciences,
Jilská 1, 110 00 Prague, Czech Republic
Email: martina.mysikova@soc.cas.cz

Abstract: Poverty indicators are sensitive to the parameters employed in their construction. We focus on the sensitivity of the income poverty headcount ratio to the parameters of equivalence scales: the weights assigned to adults and children in a household. The design of equivalence scales considers economies of scale, however, poverty estimates may be sensitive to the scale's parameters. In addition, tracking progress in alleviation of poverty can be challenging: for one set of parameters, the data can suggest a decrease, while another sees an increase in poverty. We first propose a practical measure of the sensitivity of an income poverty indicator to OECD-type equivalence scales. We next derive a simple methodology to assess the robustness of income poverty rate comparisons to the parameters of the scales in two points in time or space. We demonstrate the methods on microdata on 32 European countries included in the EU-SILC, employing the income headcount poverty index. Our study finds that a 1-percentage-point change in the weight of adults is associated with a 0.14 to 0.85% change in the income poverty headcount rate. Sensitivity to the weight of children is somewhat lower, between 0.06 and 0.44%. We further show that the robustness of poverty rates in 2019 data compared to 2018 exceeds 90% in 14 of 32 countries. In interval estimates of poverty rates, measurements for 13 of 32 countries are completely non-robust. This paper contributes to the ongoing debate on assessments of the robustness of poverty measures. High sensitivity of income poverty indicators to equivalence scales and/or low robustness of comparisons of poverty indicators over time and space can ultimately have negative consequences on policies.

Keywords: Income poverty; Equivalence scales; Sensitivity; Robustness

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I. INTRODUCTION

Income poverty indices are commonly used in official reporting of poverty data and tracking progress in alleviation of poverty. In some cases, they are reported as individual indicators, while in others, they are presented within sets of various poverty indices, or are included as one dimension of multidimensional poverty indicators. In either case, conceptualization of income poverty measurement is based on two main steps: identification of the poor, and aggregation of poverty information. This primarily requires a researcher to define income, to determine the units of reporting (e.g., individuals or households), and to establish a poverty line. The definition of income is determined by identifying income which is received either at the individual or household level. An equivalence scale is then chosen and used to transform the total household income into comparable units. In practical applications, some of the parameters in income poverty rate estimation procedures are chosen arbitrarily (Hagenaars et al., 1994).

In this paper, we focus on a single step in the process: selection of the equivalence scale. Equivalence scales are used to determine an equivalised household size and the associated equivalised income, i.e., household income per equivalent unit (typically a single adult household). The rationale behind determining equivalised income lies in adjusting the household size, considering economies of scale. Normalizing household income using an equivalence scale should "*assure that two people with the same equivalent income are equally well off*" (Ravallion, 2016, p. 168). Nevertheless, economies of scale from living together (per-capita income) and maximum economies of scale (total household income) are also considered in practical applications. Haughton and Khandker (2009) even argue that consumption per capita is the most commonly used measure of welfare, and that "*equivalence scales are not widely used because of the difficulty of agreeing on an appropriate set of weights*" (p. 83).

Nevertheless, the existence of economies of scale has been widely accepted by the literature (van der Gaag and Smolensky, 1982; Hagenaars et al., 1994). Literature recognizes numerous approaches to deriving equivalence scales: while the most widely used scales in practical applications are traditionally based on consumption expenditures, also referred to as a behavioural approach (Deaton, 2018), subjective perceptions of minimum income needed to make ends meet (Kapteyn and van Praag, 1976; Bishop et al., 2014; Mysíková et al., 2021), income satisfaction (Bütikofer and Gerfin, 2009), social security equivalence scales (Hagenaars, de Vos and Zaidi, 1994), and expert opinions (Buhmann et al., 1988) have all been applied. Clearly, different equivalence scales can result in different estimations of a poverty rate (Buhmann et al., 1988). Traditionally, the construction of an equivalence scale either does or does not distinguish between separate weights for adult and child household members (McClements, 1977). However, generally, there is no consensus on methods for calculating the relative weights (costs) of children or for household economies of scales (Deaton and Zaidi, 2002) and, as further argued by Ravallion (2016), "*setting equivalence scales remains one of the most difficult steps in applied welfare measurement*" (p. 172).

The ambiguous views on the adoption of different types of equivalence scales in poverty rates estimations raise questions such as whether one scale, defined by a set of parameters, is superior to another with a different set of parameters. Consequently, for practical applications it is reasonable to assess the sensitivity of income poverty rate estimates to different equivalence scales. Numerous authors have explored the sensitivity of income poverty indices comparisons to equivalence scales (e.g., Buhmann et al., 1988; Lanjouw and Ravallion, 1995; Batana et al. 2013; Ravallion, 2015). In particular, assessments of country orderings based on income poverty indices (Ravallion and Bidani, 1994; Duclos and Makdissi, 2005) have been common.

We contribute to this stream of literature by examining the sensitivity of the income poverty headcount ratio to equivalence scales and by assessing the robustness of over-time and across-space comparisons of income poverty indices to parameters of equivalence scales. We focus on the OECD-type equivalence scale and present two simple measures. We first propose a way to measure the sensitivity of the income poverty indicator to an equivalence scale. The measure reflects the average change in the income poverty indicator related to a unit change in the weight assigned to an additional adult or child in the household (two separate measures are reported). This substantially distinguishes our paper from previous research. The approach we propose allows us to quantify the mean sensitivity of income poverty indicator to equivalence scales using two single values. In contrast, sensitivity analyses employed in prior studies are usually based on reporting different values of an income poverty indicator corresponding to different equivalence scales and commenting on the differences.

Further, an indicator's sensitivity to the equivalence scale can change over time and across (geographical) space, and the sensitivity measure does not answer the question of how robust the comparisons will remain. To address this, we apply a novel approach to assessment of the robustness of comparisons of income poverty indices to parameters of equivalence scales. The measure is based on comparisons of income poverty indicators in two points in time (space), and assesses the share of combinations leading to the same outcome (an increase or a decrease in the indicator value) when scale parameters (adult/child weights) are changed. A low level of robustness of comparisons of the metric's values over time or across spatial units can raise questions about the validity of such comparisons. This is particularly important in exercises such as tracking progress in poverty alleviation or assessing the effectiveness of anti-poverty policies. If one set of parameters suggests that the poverty level decreased while another set suggests the reverse, clearly the comparison would provide ambiguous conclusions.

Though the terms *sensitivity* and *robustness* are related, to avoid confusion, throughout the paper we refer to *sensitivity* when we examine the sensitivity of an income poverty indicator to an equivalence scale at a given point in time and space; and to *robustness* when we assess the robustness of comparisons of poverty indicator values in two points across time or space. We demonstrate the approach empirically, examining the sensitivity and robustness of comparisons of the income poverty headcount ratio to equivalence scales using European Union (EU) member states as a case study for the empirical exercise.[2] Official data on income poverty and income distribution in the EU are based on equivalised income using the OECD-modified scale, which makes it appropriate for the demonstration of our approach.

II. LITERATURE REVIEW

For decades, researchers have been interested in how sensitive poverty and inequality measures - and income distribution measures in general - are to the parameters of equivalence scales. A seminal paper on the sensitivity of poverty indicators to equivalence scales was published by Buhmann et al. (1988), who explored ten countries using Luxembourg Income Study (LIS) data. They reviewed dozens of previously published studies, classifying them into categories: expert

[2] Note that, in addition to data on EU member states, the data set includes information for the following countries: Iceland, Norway, Serbia, and Switzerland, which are also included in our analyses. Also note that the UK officially left the EU on 31 January 2020, so the results in this work refer to the period when the UK was an EU member country.

statistical (e.g., OECD scales), expert programme (e.g., based on defining benefits for social programmes), consumption, and subjective scales. They adopted an approach that allowed them to represent the scales by a single parameter and showed that the choice of equivalence scale can systematically affect both absolute and relative measures of poverty levels. Atkinson et al. (1995) later updated the analyses performed by Buhmann et al. (1988), primarily aiming to outline a basis for selecting equivalence scales for the purposes of exploring personal income distribution in OECD countries and changes in the distribution in the mid-1980s.[3] Based on these analyses, they decided to utilize per capita and square root scales.

Other examples of sensitivity analyses comparing the results of a few standard equivalence scales published in the past decades have been provided in different national contexts including EU countries (O'Higgins and Jenkins, 1990; de Vos and Zaidi, 1997); the United States (Cutler and Katz, 1992); Canada (Phipps, 1993); comparing the United States and Germany (Burkhauser et al., 1996); and groups of developing countries (Batana et al., 2013; Newhouse et al., 2017). Wider sets of equivalence scales have been examined in different national contexts including the UK (Cowell and Jenkins, 1992a); the UK and Spain (Duclos and Mercader-Prats, 1999); the United States (Short et al., 1999; Folbre et al., 2018); and the Russian Federation (Abanokova et al., 2020).

Recently, concerns about the sensitivity of poverty estimates to equivalence scales have been raised by - among others - Ferreira and Ravallion (2008), who outlined methodological challenges of comparing global poverty estimates including concerns regarding the sensitivity of results. Batana et al. (2013) reacted to these challenges and showed that, if alternative equivalence scales are adopted instead of per capita income/consumption, the poverty estimates drop from over 30% to between 3 and 13%; arguing that the figures provided by Chen and Ravallion (2010) were substantially overestimated. Ravallion (2015), however, responded that the seeming overestimation of estimates suggested by Batana et al. (2013) was a result of testing sensitivity without calibrating the reference poverty line. Keeping this in mind, Newhouse et al.

[3] The paper by Buhmann et al. (1988) was primarily empirical. Coulter et al. (1992) built on the work by Buhmann et al. (1988), and presented theoretical results on the dependency of inequality/poverty metrics on equivalence scale relativities; utilizing 1986 UK Family Expenditure Data, they empirically illustrated the theoretical results. Banks and Johnson (1994), however, raised concerns relating to the stability of the results reported by Coulter et al. (1992), suggesting that using datasets from different years and a more general representation of the equivalence scale could potentially lead to different results. In their reaction, Jenkins and Cowell (1994) were able to demonstrate that the criticism by Banks and Johnson (1994) was unjustified.

(2017) estimated child poverty rates and examined their sensitivity to alternative equivalence scales. Unlike Batana et al. (2013), they chose a pivot household resembling a "typical household" as suggested by Ravallion (2015).

The OECD equivalence scale introduced by OECD (1982) is central to our study.[4] The original OECD-scale assigns a value of 1 to the first household member, 0.7 to each additional adult and 0.5 to each child; while the OECD-modified scale assigns relative weights of 0.5 and 0.3, respectively.[5] The parameters of both the original and the modified OECD equivalence scales are arbitrarily chosen and their main purpose – to adjust the household income to account for economies of scale – may be limited. For this reason, a high degree of sensitivity in an income poverty metric to parameters of the equivalence scale may render the validity of comparisons of poverty metrics values over time or across spatial units questionable.

From the perspective of our empirical application, we base our work on previously published studies utilizing the same data sets. Hagenaars et al. (1994) were amongst the first to rigorously assess the sensitivity of a headcount poverty ratio to three alternative equivalence scales (original OECD, OECD-modified, and subjective equivalence scales) in twelve EU countries. They focused on choosing an equivalence scale in the context of cross-country comparisons, rather than on the choice of a scale for a particular country. According to their analyses, the share of poor households was slightly lower when the original OECD scale was used, compared to the results using the modified scale. More recent assessments of the influence of alternative equivalence scales on poverty/inequality comparisons have been included in studies by Tóth and Medgyesi (2011), who examined the differences in patterns of income distributions in the new and old EU member states, and Kuypers and Marx (2018), who focused on income-wealth

[4] For international comparisons, OECD currently uses a "square-root equivalence scale", i.e., the equivalent size of the household is determined as the square root of household size, which does not distinguish between adults and children (OECD, 2020).

[5] Hagenaars et al. (1994) analysed the European Community Household Panel (ECHP) data and suggested that the original OECD scale placed relatively heavy weight on additional persons. As a result, they proposed an OECD-modified scale. Dennis and Guio (2004) argued that the switch from the original to the OECD-modified scale in the EU was a consequence of the decreasing proportion of food expenditure in household budgets implying more economies of scale. The (political) decision was formalized in collaboration with the EU member states in 1998. This scale has since been used by Eurostat for equivalising household income within the EU-SILC survey, an official source of data on income and living conditions in the European Union (European Commission, 2003). Hagenaars et al. (1994) however, queried whether, in cross-country comparisons, a single equivalence scale for all EU countries should be applied, or whether a single methodology to estimate the scales, but with (potentially) different scales applied across countries would be a better approach.

poverty in selected EU countries. In their recent studies, Mysíková and Želinský (2019) and Mysíková (2021) explored the sensitivity of income poverty rates to different combinations of adult and child weights within the OECD-type equivalence scale, utilizing the coefficient of variation as a simple indicator of sensitivity.

Overall, we primarily build our approach on studies examining the sensitivity of income poverty indicators to equivalence scales, considering a wide range of parameter values – thought of as a grid of combinations (Duclos and Mercader-Prats, 1999; Batana et al., 2013; Abanokova et al., 2020). These studies also serve as a basis for the visual representation of results presented throughout the paper. The measure of sensitivity introduced in this study is based on the intuition presented by Želinský and Mysíková (2019), but we develop a rigorous methodology allowing interpretations in terms of average marginal changes. Our approach addresses concerns raised by Ravallion (2015) about a study by Batana et al. (2013), which did not calibrate the reference poverty line. Results reported in this study are based on the Eurostat methodology (Eurostat, 2018a), within which such concerns are irrelevant because a new poverty line is set for each combination of parameters of equivalence scale. As suggested by this literature review, previous research has addressed assessments of the sensitivity of poverty rates to equivalence scales, but little attention has been given to exploration of the robustness of poverty rates comparisons to equivalence scales over time or across spatial units. Ultimately, we contribute to existing literature by examining the sensitivity and robustness of income poverty rates and comparisons to the parameters of equivalence scales.

III. METHODOLOGY

In this section we describe both measures utilised in our study. We first introduce the methodological background of the OECD-type equivalence scales. We next derive the measure of sensitivity of the income poverty indicator to parameters of the equivalence scale, which are the weights assigned to adults and children in the household; and the measure of the robustness of income poverty indicator comparisons over time and across spatial units. We conclude this section with a description of the data used for an empirical application of our approach.

3.1 Methodological background

Parametrization of the equivalent size of a household (E) can be expressed as $E = S^e$ (Buhmann et al., 1988) or $E = (A + cK)^e$ (Cutler and Katz, 1992), where $S = A + K$ is the household size (number of household members), A is the number of adults, K the number of children, c is a constant reflecting the resource cost of a child relative to that of an adult, and e is an indicator of the degree of overall economies of scale within the household (Duclos and Araar, 2006).

In line with Buhmann et al. (1988), Cutler and Katz (1992) and Duclos and Araar (2006), equivalence scales in general can be given as:

$$E = \left(1 + c_a\left(A - 1\right) + c_k K\right)^e \,, \tag{1}$$

where: A is the number of adults; K the number of children; c_a is a constant reflecting the resource cost (also referred to as relative weight or discount factor) of second and other adults relative to that of the first adult in the household; c_k is a constant reflecting the resource cost of a child relative to that of the first adult in the household; and e is an economies of scale factor.

The general construction of an equivalence scale given by Equation (1) involves various special cases of equivalence scales, including the original and modified OECD scales. In the OECD-type scales, the indicator of the degree of economies of scale within the household (e) has a value of 1, i.e., there is no explicit recognition of economies of scale built into the construction of the scale. However, as noted by Citro and Michael (1995), the economies of scale are built into the "discount" for the additional household members, i.e., in the relative adult and child weights c_a and c_k.

The OECD-modified equivalence scale ($e = 1$, $c_a = 0.5$, $c_k = 0.3$ in Equation (1)) is utilized to equivalise income in the EU within the European Union Statistics on Income and Living Conditions (EU-SILC) survey.[6] Other equivalence scales, which are also special cases of

[6] The OECD-modified equivalence scale has also been adopted for equivalising income in Australia within the Household, Income and Labour Dynamics in Australia (HILDA) survey (Wilkins et al., 2020). In the USA, the construction of equivalence scales is based on Equation (1);they adopt a three-parameter scale (US Census Bureau, 2016): $E = A^{0.5}$ for $A \in \{1; 2\}$ & $K = 0$; $E = (A + 0.8 + 0.5(K – 1))^{0.7}$ for single parents & $K \geq 1$; and $E = (A + 0.5K)^{0.7}$ for other families; where A is the number of adults, and K is the number of children.

Equation (1), commonly used in practical applications include square root scales (e = 0.5, c_a = 1, c_k = 1) currently applied by the OECD for adjusting income (OECD, 2020). International comparisons of income poverty measures for the developing world conducted by the World Bank are usually based on per capita household income or consumption expenditures (e = 1, c_a = 1, c_k = 1 in Equation (1)), which ignore economies of scale (World Bank, 2020).

3.2 The measure of the sensitivity of the income poverty indicator to OECD-type equivalence scales

We propose a simple approach to assessment of the sensitivity of an income poverty indicator to parameters (weights assigned to adults and children in the household). We describe the approach as applied to the headcount income poverty index (H), but the approach can be applied to various indices of income poverty. We utilize an OECD-type equivalence scale, which is a special case of Equation (1) with an economies of scale factor e = 1. Adopting the terminology used by Eurostat, constants c_a and c_k in Equation (1) reflect the resource costs of relative adult and child weights, respectively, and we use the notation w_a and w_k. With e = 1, and the Eurostat terminology, Equation (1) simplifies to the equivalent size of a household S_E:

$$S_E = 1 + w_a(A - 1) + w_k K \, , \tag{2}$$

where A is the number of adults in the household, K is the number of children in the household, w_a is the weight assigned to second and other adult members in the household, and w_k is the weight assigned to children in the household.

Our approach is based on modelling the distribution of H as a function of adult and child weights. In the first step, using the Eurostat methodology (see Section 3.4), we estimate the values of H for each combination of adult and child weights on a given grid of weights. Specifically, we use a grid of all possible combinations of adult and child weights; i.e., elements of vectors of sequences $\mathbf{w}_a = \mathbf{w}_k = \left\{ \frac{l}{N} \right\}_{l=0}^{N}$, where N is the number of non-zero elements of the sequence of weights. In our empirical application we use N = 100. Clearly, a larger N is associated with greater precision, however, using N = 1000 in the empirical application leads to negligible differences from the results based on N = 100. In the second step, in order to smooth the surface representing the relationship between H, w_a and w_k; we use the generalized additive models with an integrated smoothness estimation procedure (Wood, 2011).

A similar intuition with visual representation of the relationship between equivalence scale parameters and the income poverty indicator was previously adopted by Duclos and Mercader-Prats (1999), Batana et al. (2013), and Abanokova et al. (2020). We build on these studies and extend the visual approach to estimating the average change in the poverty indicator associated with a unit change in equivalence scale parameters (adult and child weights).

Once we have smoothed the values of H for each value of the adult-child weight combinations across the whole grid, we can estimate the degree of the sensitivity of H to weights. The construction of our sensitivity measure is based on the idea of average marginal changes. To find the sensitivity measure of H to the adult weight, we apply the following intuition: For the j^{th} value of w_k , $j \in \{0; 1; \ldots; N\}$ we calculate the change in H for each unit change in w_a:

$$\delta_{l+1}(w_k)_j = \frac{\Delta H_{l+1}}{\Delta w_{a_{l+1}}}, \; l \in \{0; 1; \ldots; N\}. \tag{3}$$

The mean change in H for each j is found as a simple average:

$$\bar{\delta}(w_k)_j = \frac{\sum_{l=1}^{N} \delta_l \, (w_k)_l}{N}. \tag{4}$$

Since the relationship between H and w_a for the given value of w_k does not necessarily have to be monotonous along all values of w_a, we consider two partial cases: the mean change of H increasing along the values of w_a, and the mean change for H decreasing along the values of w_a, denoted as

$$\bar{\delta}^{+}(w_k)_j = \frac{\sum_{l=1}^{N^{+}} \delta_l^{+}(w_k)_l}{N^{+}} \text{ and} \tag{5}$$

$$\bar{\delta}^{-}(w_k)_j = \frac{\sum_{l=1}^{N^{-}} \delta_l^{-}(w_k)_l}{N^{-}}, \text{ respectively.} \tag{6}$$

Having found the partial changes $\delta(w_k)_j$ for each value of w_k, we can calculate the measure of sensitivity β constructed as an average absolute change of $\delta(w_k)_j$ across all values of w_k:

$$\beta_A = \frac{\sum_{j=1}^{N} \left| \bar{\delta}(w_k)_j \right|}{N}. \tag{7}$$

In addition to the overall measure of sensitivity, it is also possible to report a mean sensitivity measure for increasing values of H:

$$\beta_A^+ = \frac{\sum_{j=1}^{N^+} \left| \bar{\delta}^+(w_k)_j \right|}{N^+} \tag{8}$$

and a mean sensitivity measure for decreasing values of H:

$$\beta_A^- = \frac{\sum_{j=1}^{N^-} \left| \bar{\delta}^-(w_k)_j \right|}{N^-}. \tag{9}$$

β_A^+ corresponds to a positive relationship between the adult weight and the value of an income poverty indicator, while β_A^- is related to a negative relationship. By positive relationship, we mean that an increase in the adult weight is associated with an increase in the value of the income poverty indicator. The negative relationship refers to a decrease in the value of the income poverty indicator associated with an increase in the adult weight.

Analogously, sensitivity measures for H sensitivity to the weight assigned to children in the household can be derived:

$$\beta_K = \frac{\sum_{j=1}^{N} \left| \bar{\delta}(w_a)_j \right|}{N}. \tag{10}$$

$$\beta_K^- = \frac{\sum_{j=1}^{N^-} \left| \bar{\delta}^-(w_a)_j \right|}{N^-} \quad (11)$$

$$\beta_K^+ = \frac{\sum_{j=1}^{N^+} \left| \bar{\delta}^+(w_a)_j \right|}{N^+} \quad (12)$$

The values of β_A and β_K can be interpreted as an average change in H as a result of a unit ($1/N$) change in the adult and child weights, respectively. The higher values of β_A and β_K are associated with greater sensitivity of H to weights. The estimated sensitivity measures can be reported in absolute (average percentage-point-change in H associated with a 0.01-point-change in the child/adult weight) and relative (average relative percentage changes in H associated with 0.01-point-change in the child/adult weight) terms. In the former, ΔH_{l+1} is defined as an absolute change: $\Delta H_{l+1} = H_{l+1} - H_l$, while in the latter as a relative change: $\Delta H_{l+1} = \frac{H_{l+1} - H_l}{H_l}$. Although we estimate both absolute and relative measures of sensitivity, in Section IV we report only the results based on the relative measure. Detail results including those based on the absolute measure can be found in the online appendix.

3.3 The measure of the robustness of income poverty indicators comparisons

The measure of the robustness of comparisons is built on the same intuition as described in Subsection 3.2. Having found the smoothed values of H for each combination of adult and child weights; i.e., elements of vectors of sequences $\mathbf{w}_a = \mathbf{w}_k = \left\{ \frac{l}{N} \right\}_{l=0}^{N}$, where N is the number of non-zero elements of the sequence of weights; we can find the differences

$$\Delta H_{T,T-t}(w_a, w_k)_i = H_T(w_a, w_k)_i - H_{T-t}(w_a, w_k)_i \, , \quad (13)$$

for each combination (pair) of values of w_a and w_k, where $H_T(w_a, w_k)_i$ is the smoothed value of H for the i^{th} combination of w_a and w_k in time T. The approach is visually demonstrated in Figure 1.

If the differences ΔH_i are positive (negative) for all $(N+1)\times(N+1)$ combinations of weights $[w_a; w_k]$, we can claim that the change in the income poverty rate is robust to weights assigned to adults/children in the household.

This intuition further allows us to construct a simple measure of the robustness of comparisons in two points in time (space):

$$R_{T,T-t} = \max\left(\frac{\sum_i \mathbf{I}(\Delta H_i > 0)}{(N+1)^2};\ \frac{\sum_i \mathbf{I}(\Delta H_i < 0)}{(N+1)^2} \right), \tag{14}$$

where $\mathbf{I}(\cdot)$ is an indicator function, and ΔH_i represents the differences given by Equation (13). The values of $R_{T,T-t}$ range between [0.5; 1], with 1 representing the full robustness and 0.5 representing the lowest possible level of robustness.

Analyses of changes in income poverty levels are usually based on point estimates. We further extend our results based on point estimates to results based on interval estimates using the methodology described in this section. In this case, Equation (13) includes estimates of standard errors (*se*):

$$\Delta H'_{T,T-t}(w_a, w_k)_i = \left[H_T(w_a, w_k)_i \pm se\right] - \left[H_{T-t}(w_a, w_k)_i \pm se\right], \tag{15}$$

with estimates of *se* estimated in accordance with Eurostat methodology using bootstrapping. In the case including interval estimates, the measure $R_{T,T-t}$ given by Equation (14) is based on non-overlapping interval estimates of H.[7] It is thus expected that, in comparison to point estimates, interval estimates will result in lower levels of robustness. Any small year-on-year increase in the value of H index will be defined as an increase (decrease) by the procedure based on point estimates, but if the corresponding interval estimates overlap, it is likely that no change will be identified by the alternative procedure.

[7] Note that comparisons based on interval estimates are based on N=20 non-zero weights (precision of 0.05) due to the highly computationally intensive nature of estimations. Nevertheless, the results based on the 0.05-precision differ from the 0.01-precision results on average by 1%, suggesting high accuracy of results.

To illustrate our approach, a graphical representation is demonstrated in Figure 1. Top panels of Figure 1 represent the smoothed surfaces of the income headcount income poverty rate as a function of adult and child weights in two consecutive periods.

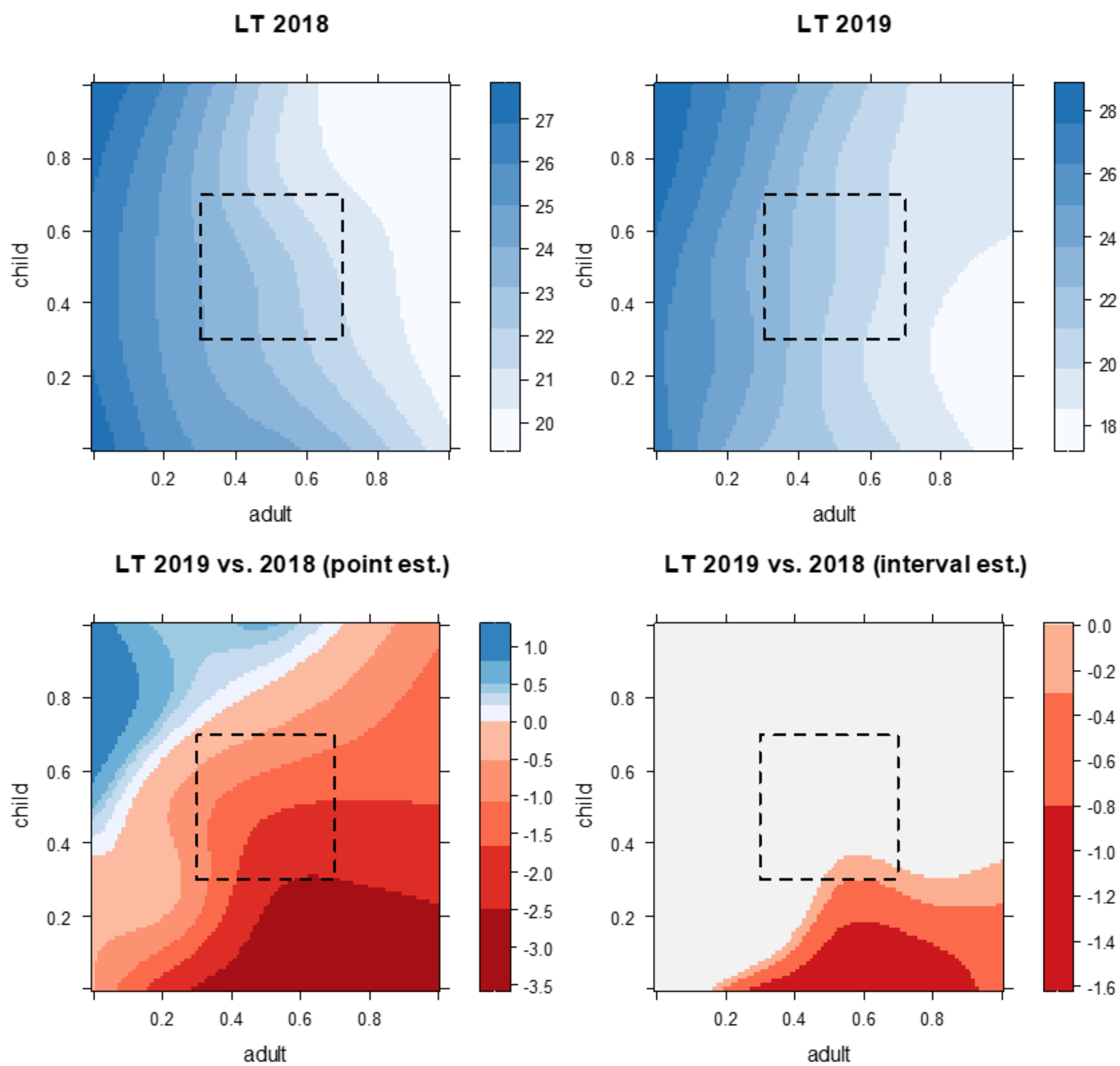

Figure 1. A visual illustration of the approach (Lithuania 2018 and 2019)

Notes: Top panels depict the distribution of *H* (point estimates) as a function of adult/child weights. Bottom panels depict the differences in *H* between 2018 and 2019 as a function of weights; point estimates appear in the left panel, interval estimates in the right panel. The dashed square corresponds to the limited range of weights [0.3; 0.7]. The grey area in the bottom right panel corresponds to adult-child weights combinations for which interval estimates of *H* overlap. Results are based on the precision of 0.01, referring to *N*=100.

Source: Author's calculations based on EU-SILC 2018-2019 data (Cross UDB, 2020-09 version).

When it comes to practical applications, such as tracking the progress of income poverty reduction over time or across spatial units in a given country, it might not always be reasonable to assess the sensitivity of an income poverty indicator to the whole range of weights. Researchers and practitioners may be interested only in limited sets of weights that may be considered for practical applications, and assess the sensitivity of income poverty indicators to weights from the limited range. We will demonstrate the assessment of the sensitivity and robustness of income poverty indicator comparisons considering limited intervals of adult and child weights arbitrarily set as $w_a \in [0.3; 0.7]$ and $w_k \in [0.3; 0.7]$. Clearly, these intervals are arbitrary, and are based on the union of weights utilized in the original ($w_a = 0.7$; $w_a = 0.5$) and modified ($w_a = 0.5$; $w_a = 0.3$) OECD equivalence scales. Scholars and practitioners may choose different sets of weights depending on the nature of their analyses.

As further illustrated in the bottom left panel, the negative differences in *H* between 2018 and 2019 are substantially more likely to occur than the positive differences. This indicates that the majority of adult-child combinations suggest a decrease in the income poverty level between 2018 and 2019 in this particular case. Limiting the weights to [0.3; 0.7] interval suggest fully robust comparisons (negative values for all combinations inside the dashed square). However, when estimations of standard errors are included in the analyses (bottom right panel), robustness levels drop considerably (the grey area represents comparisons for which interval estimates overlap, suggesting inconclusive results).

3.4 Description of data

The official European statistics on poverty rely on data from the household survey Statistics on Income and Living Conditions (EU-SILC). The survey is compulsory for EU member countries, has been conducted annually since 2005, and is harmonized by Eurostat. The data are collected and reported at both household and individual levels, and all household members aged 16+ are interviewed. The analyses in this study are based on EU-SILC 2015–2019 household survey microdata (Cross UDB, version of 2020-03) for 32 countries included in the survey (28 EU member states + Iceland, Norway, Serbia, and Switzerland). According to the Eurostat methodology (Eurostat, 2018b), total household disposable income includes all monetary incomes received from any source by each member of a household; these include income from work, allowances and social transfers including pensions, rental income, profit from capital investments plus any other household income; disposable income is net of taxes and social contributions that have been paid.

The national income poverty line is expressed as 60% of the median of equivalised disposable income and the corresponding income poverty headcount ratio (*H*), also referred to as at-risk-of-poverty rate in the Eurostat terminology, is defined as the proportion of people living in households with equivalised disposable income below the poverty line (Eurostat, 2018a). Results are weighted by the personal cross-sectional weight, so that the income poverty rate expresses the share of people, not households, below the poverty line.

IV. RESULTS AND DISCUSSION

In this section we describe and discuss results based on the approach proposed in this study. The first section of results refers to the measures of sensitivity of the income poverty headcount ratio to the weight assigned to second and other adults in a household (β_A) and to the weight assigned to children in a household (β_K). We first explore the overall sensitivity and then proceed to its disaggregation, which differentiates the positive and the negative relationships between the weights and the income poverty indicator. These sensitivity analyses are informative about how sensitive income poverty indicators are to adopting different sets of adult/child weights in the construction of equivalence scales in one point in time. However, they do not provide answers to questions about whether comparisons over time are robust, or whether we can, e.g., claim that comparisons of income poverty rates between two periods are robust to different sets of weights assigned to parents and children. Assessments of the robustness of such comparisons are presented in the second section of results, which is based on the measure of the robustness of the income poverty headcount ratio comparisons over time. In both cases, in addition to the results based on the whole range of weights (i.e., between 0 and 1), we also discuss the results based on a limited range (between 0.3 and 0.7).

Table 1. Relative sensitivity of income poverty rates to adult and child weights [2019, in %]

Country	Whole range		Limited range	
	Adult weight (1)	Child weight (2)	Adult weight (3)	Child weight (4)
Austria	0.440	0.452	0.388	0.602
Belgium	0.418	0.130	0.453	0.092
Bulgaria	0.310	0.059	0.359	0.053
Croatia	0.343	0.135	0.284	0.105
Cyprus	0.531	0.138	0.564	0.083
Czechia	0.855	0.414	1.099	0.186
Denmark	0.480	0.372	0.424	0.208
Estonia	0.348	0.216	0.297	0.126
Finland	0.724	0.409	0.877	0.227
France	0.320	0.262	0.169	0.353
Germany	0.349	0.193	0.337	0.167
Greece	0.232	0.194	0.098	0.225
Hungary	0.488	0.373	0.353	0.521
Iceland*	0.816	0.557	0.752	0.588
Ireland*	0.563	0.372	0.801	0.383
Italy*	0.147	0.139	0.042	0.218
Latvia	0.264	0.163	0.242	0.117
Lithuania	0.417	0.104	0.480	0.061
Luxembourg	0.252	0.267	0.159	0.343
Malta	0.451	0.094	0.534	0.041
Netherlands	0.469	0.322	0.327	0.251
Norway	0.577	0.338	0.659	0.220
Poland	0.332	0.199	0.307	0.196
Portugal	0.270	0.129	0.162	0.044
Romania	0.161	0.063	0.100	0.054
Serbia	0.145	0.058	0.062	0.054
Slovakia	0.579	0.346	0.556	0.466
Slovenia	0.687	0.331	0.671	0.187
Spain	0.145	0.105	0.107	0.103
Sweden	0.446	0.162	0.563	0.120
Switzerland	0.301	0.260	0.270	0.274
United Kingdom*	0.259	0.226	0.224	0.297

Notes: The table reports the relative sensitivity of the poverty headcount ratio to adult and child weights. Columns 1 and 2 report the measure of sensitivity based on the whole range of weights, i.e., the interval [0; 1] for both adult and child weights. Columns 3 and 4 report the measure of sensitivity on a limited range of weights, i.e., the interval [0.3; 0.7] for both adult and child weights.

* 2019 data not available, 2018 results reported.

Source: EU-SILC 2018-2019 (Cross UDB, version of 2020-09).

4.1 Overall sensitivity of the headcount income poverty index to adult and child weights

According to the most recent (2019) data (Table 1), in half of the EU countries, the relative measure of the sensitivity of the income poverty headcount ratio to the adult weight exceeds the value of 0.4. This value can be interpreted in this way: a 0.01-point change in the adult weight is associated with an average 0.4-percentage change in the value of the income poverty headcount ratio. The relative sensitivity of income poverty rate to the adult weight ranges between 0.145 (Spain and Serbia) and 0.855 (Czechia), while the relative sensitivity to the child weight ranges between 0.058 (Serbia) and 0.557 (Iceland). The relative sensitivity of the income poverty headcount ratio to the child weight is generally lower than to the adult weight, with values exceeding 0.2 in half of the countries. These results are in accordance with findings that Abankova et al. (2020) reported for Russia.

Assigning extremely low or extremely high weights to either group of household members may not be reasonable under certain conditions. In practical applications, it may be more reasonable to perform sensitivity analyses on a limited range of weights. We demonstrate this exercise using a limited range between 0.3 and 0.7 for both adult and child weights (columns 3 and 4 of Table 1). The 2019 data suggests that the variation in the relative sensitivity measure based on a limited range of weights is mostly greater than in the case of the whole range of weights. For example, the measure of relative sensitivity to the weight assigned to adults increases from 0.855 to 1.099 in the Czech subsample, while it decreases from 0.147 to 0.042 according to the Italian data.

In addition to the main results reported in Table 1, we also performed a robustness assessment of results utilizing different age definitions for adults. The official approach adopted by Eurostat defines adult household members as those aged 14+. As noted by Hagenaars et al. (1994, p. 16), the definition of adults in the OECD family equivalence scale was “a rather arbitrary choice”. We compared the sensitivity measures based on the official approach reported in Table 1 to estimates based on alternative thresholds of 15, 16, 17, and 18 years of age; the results suggest strong correlations between the official and alternative thresholds. The values of coefficients of correlations are greater than 0.95 for the measure of sensitivity to the child weight; and 0.98 for the measure of sensitivity to the adult weight. Note that a more detailed assessment of sensitivity related to the age definition of adults is beyond the scope of this paper. Please refer to Table S.A in Supplementary Materials for more detailed results over the whole period of 2015 to 2019.

Table 2. Descriptive statistics of the relative sensitivity measure (2015 – 2019 data)

Country	Adults				Children			
	Mean (1)	Min (2)	Max (3)	CV (4)	Mean (5)	Min (6)	Max (7)	CV (8)
Austria	0.413	0.354	0.457	0.051	0.394	0.327	0.462	0.104
Belgium	0.377	0.340	0.418	0.069	0.163	0.116	0.242	0.198
Bulgaria	0.263	0.232	0.328	0.111	0.060	0.050	0.069	0.091
Croatia	0.300	0.262	0.343	0.084	0.123	0.092	0.149	0.163
Cyprus	0.369	0.245	0.531	0.205	0.162	0.113	0.237	0.223
Czechia	0.804	0.736	0.871	0.060	0.438	0.354	0.549	0.112
Denmark	0.487	0.402	0.559	0.078	0.436	0.336	0.541	0.135
Estonia	0.418	0.335	0.479	0.117	0.248	0.198	0.303	0.105
Finland	0.673	0.617	0.724	0.040	0.450	0.355	0.554	0.120
France	0.333	0.302	0.361	0.058	0.341	0.262	0.412	0.121
Germany	0.341	0.319	0.373	0.047	0.201	0.163	0.243	0.103
Greece	0.161	0.106	0.235	0.257	0.196	0.165	0.237	0.093
Hungary	0.384	0.305	0.488	0.131	0.365	0.237	0.502	0.194
Iceland*	0.829	0.728	0.915	0.063	0.625	0.517	0.751	0.113
Ireland*	0.437	0.285	0.642	0.211	0.289	0.204	0.432	0.250
Italy*	0.149	0.106	0.184	0.156	0.160	0.127	0.197	0.113
Latvia	0.309	0.214	0.407	0.226	0.150	0.113	0.189	0.161
Lithuania	0.356	0.291	0.433	0.126	0.100	0.072	0.123	0.127
Luxembourg	0.273	0.228	0.319	0.092	0.319	0.267	0.394	0.094
Malta	0.432	0.405	0.462	0.046	0.112	0.062	0.161	0.288
Netherlands	0.457	0.395	0.532	0.076	0.335	0.264	0.429	0.126
Norway	0.595	0.569	0.629	0.030	0.402	0.299	0.499	0.130
Poland	0.307	0.244	0.373	0.127	0.200	0.164	0.258	0.130
Portugal	0.248	0.219	0.274	0.070	0.136	0.099	0.187	0.131
Romania	0.149	0.117	0.182	0.130	0.102	0.063	0.128	0.168
Serbia	0.111	0.079	0.150	0.225	0.053	0.036	0.065	0.158
Slovakia	0.533	0.452	0.579	0.065	0.368	0.298	0.461	0.118
Slovenia	0.561	0.456	0.692	0.151	0.238	0.172	0.400	0.297
Spain	0.142	0.117	0.156	0.077	0.092	0.043	0.129	0.240
Sweden	0.438	0.408	0.466	0.038	0.221	0.158	0.326	0.227
Switzerland	0.336	0.277	0.405	0.112	0.310	0.258	0.369	0.104
United Kingdom*	0.288	0.259	0.332	0.075	0.302	0.226	0.386	0.147

Notes: The table reports descriptive statistics of the relative sensitivity measure over the period of 2015–2019 (yearly observations). The measures of sensitivity are based on the whole range of weights, i.e., the interval [0; 1].
* indicates that 2019 data was not available, and 2014-2018 results are reported
Source: EU-SILC 2014–2019 (Cross UDB, 2020-09 version).

Changes in income levels as well as changes in household structure (composition and age) can result in changes in the values of the sensitivity measure. In order to explore the volatility of sensitivity to weights over time, Table 2 shows basic descriptives of the relative sensitivity measure over 2015 –2019 (mean, minimum, and maximum values, coefficient of variation). We only report the results based on the whole range of weights, whilst results based on the limited ranges can be found in the online Appendix.

The results in Table 2 confirm that, based on the mean sensitivity indices for adult and child weights over time, the income poverty ratio is generally more sensitive to the adult weight than to the child weight. On the other hand, the relatively low sensitivity to child weight is less stable over time, as suggested by a higher coefficient of variation (CV) of the relative sentitivity measure for child weights than for adult weights. The CV exceeds 10% only in 14 of 32 countries for the sensitivity measure to the adult weight, while this holds in 29 countries for the sensitivity measure to the child weight (in the remaining three countries the values exceed 9%). Overall, the CV values vary between 3% (Norway) and 25.7% (Greece) for the adult weight and 9.1% (Bulgaria) and 29.7% (Slovenia) for the child weight.

The results reported in this study demonstrate that the levels of sensitivity of income poverty rates to adult/child weights differ across countries. In principle, these results provide evidence that income poverty rates are sensitive to weights, suggesting that the use of different equivalence scales will likely result in different estimates of income poverty rates. This is particularly important when conducting comparisons, such as tracking poverty alleviation in a given country.

4.2 Partial sensitivity of the headcount income poverty index

Results reported in Table 1 demonstrate the overall relative mean sensitivity of income poverty rates to changes in weights, but they do not provide any information on the direction of the relationship. In practical applications it may not be sufficient to know how sensitively an income poverty idicator reacts to changes in weights. We may also be interested whether changes in weights are more likely to be associated with an increase or a decrease in the values of the income poverty indicator. To further understand the sensitivity of income poverty rates to weights, we report partial sensitivy measures which are estimated separately for a positive and a negative relationship. These partial sensitivity measures, β_A^+, β_A^- and β_K^+, β_K^-, are defined by Equations (8)–(9) and (11)–(12) and are reported in Tables 3 (adult weight) and 4 (child weight).

Table 3. Partial sensitivity to adult weight measures and proportions on which estimates are based [2019]

Country	Partial sensitivity measures				Proportions on which estimates are based			
	Whole range		Limited range		Whole range		Limited range	
	β_A^+	β_A^-	β_A^+	β_A^-	β_A^+	β_A^-	β_A^+	β_A^-
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Austria	0.051	–0.487		–0.388	0.099	0.901	0	1
Belgium	0.108	–0.500		–0.453	0.208	0.792	0	1
Bulgaria		–0.310		–0.359	0	1	0	1
Croatia	0.029	–0.365		–0.284	0.061	0.939	0	1
Cyprus	0.046	–0.577		–0.564	0.083	0.917	0	1
Czechia	0.026	–0.867		–1.099	0.014	0.986	0	1
Denmark	0.104	–0.556	0.014	–0.428	0.160	0.840	0.008	0.992
Estonia	0.012	–0.352		–0.297	0.013	0.987	0	1
Finland	0.010	–0.730		–0.877	0.008	0.992	0	1
France	0.073	–0.374		–0.169	0.113	0.887	0	1
Germany	0.063	–0.367		–0.337	0.050	0.950	0	1
Greece	0.102	–0.325	0.065	–0.117	0.402	0.598	0.365	0.635
Hungary	0.186	–0.636	0.098	–0.397	0.299	0.701	0.156	0.844
Iceland*	0.115	–0.850		–0.752	0.038	0.962	0	1
Ireland*	0.117	–0.590		–0.801	0.061	0.939	0	1
Italy*	0.067	–0.229	0.028	–0.039	0.433	0.567	0.415	0.585
Latvia		–0.264		–0.242	0	1	0	1
Lithuania		–0.417		–0.480	0	1	0	1
Luxembourg	0.111	–0.337	0.039	–0.197	0.381	0.619	0.219	0.781
Malta	0.077	–0.485		–0.534	0.072	0.928	0	1
Netherlands	0.206	–0.568	0.025	–0.342	0.251	0.749	0.049	0.951
Norway	0.024	–0.593		–0.659	0.026	0.974	0	1
Poland	0.054	–0.375		–0.307	0.132	0.868	0	1
Portugal	0.086	–0.332	0.008	–0.164	0.250	0.750	0.012	0.988
Romania	0.065	–0.182		–0.100	0.172	0.828	0	1
Serbia	0.035	–0.180	0.024	–0.077	0.230	0.770	0.347	0.653
Slovakia	0.113	–0.665		–0.556	0.143	0.857	0	1
Slovenia		–0.687		–0.671	0	1	0	1
Spain	0.069	–0.180	0.016	–0.120	0.314	0.686	0.100	0.900
Sweden		–0.446		–0.563	0	1	0	1
Switzerland	0.024	–0.337	0.017	–0.297	0.105	0.895	0.088	0.912
United Kingdom*	0.112	–0.282		–0.224	0.107	0.893	0	1

Notes: The table reports the measure of relative sensitivity of the poverty headcount ratio to the adult weight distinctly estimated for positive (β^+) and negative (β^-) changes, and the proportions of marginal positive (β^+) and negative (β^-) changes on which the associated measures of relative sensitivity are estimated. Columns 1–2 and 5–6 are based on the whole range of weights, i.e., the interval [0; 1]. Columns 3–4 and 7–8 are based on a limited range of weights, i.e., the interval [0.3; 0.7].
* indicates that 2019 data was not available, and 2018 results are reported
Source: EU-SILC 2018-2019 (Cross UDB, 2020-09 version).

The 2019 data clearly suggests a greater prevalence of a negative relationship between the income poverty indicator and adult/child weights. The partial sensitivity measures suggest that an increase in the adult weight is considerably more likely to be associated with a decrease in the value of the income poverty indicator (Table 3). The measures of sensitivity β_A^+ and β_A^- corresponding to a positive and negative relationship between the adult weight and the value of the income poverty indicator range between 0.010 (Finland) and 0.206 (the Netherlands) and –0.180 (Spain) and –0.867 (Czechia). Further, in some countries, the measure of sensitivity β_A^+ corresponding to a positive relationship between the adult weight and the value of the income poverty indicator is not reported, meaning that the income poverty rate always decreases with the adult weight, holding the child weight fixed. As reported in columns (3) and (4) in Table 3, within the limited range of adult weight (between 0.3 and 0.7), the income poverty headcount ratio is very unlikely to increase if the adult weight increases within the range.

The results reported in columns (3) and (4) in Table 3 still do not provide full knowledge of the relationship, as they are not informative about the relative frequency of positive (β^+) and negative (β^-) changes. They do not show the frequency of cases on which the partial measure of sensitivity is based. To demonstrate the importance of reporting not only the values of partial measures of relative sensitivity, but also the proportions of positive/negative changes, we consider the UK. According to columns (1) and (2) in Table 3, a unit increase in the adult weight is sometimes associated with a decrease in the income poverty headcount index (by 0.282% on average), and sometimes with an increase (by 0.112% on average). However, as suggested by columns (5) and (6) in Table 3, the negative relationship is identified in 89.3% of all cases, while the positive relationship only in 10.7%. Similarly to the UK, the frequency of negative marginal changes dominates across all countries; with the lowest relative frequency of roughly 56.7% in Italy.

Regarding the child weight presented in columns (1) through (4) in Table 4, the effects are more heterogeneous than the changes associated with adult weights. For child weights, the results do not suggest whether a unit change in the weights is more likely to be associated with an increase or a decrease in the values of the income poverty indicator. The frequency of negative changes ranges between 5% and 81.8%, and between 19.2% and 95% for positive changes.

Table 4. Partial sensitivity to child weight measures and proportions on which estimates are based [2019]

Country	Partial sensitivity measures				Proportions on which estimates are based			
	Whole range		Limited range		Whole range		Limited range	
	β_K^+	β_K^-	β_K^+	β_K^-	β_K^+	β_K^-	β_K^+	β_K^-
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Austria	0.498	–0.295	0.602		0.771	0.229	1	0
Belgium	0.125	–0.093	0.077	–0.065	0.599	0.401	0.631	0.369
Bulgaria	0.061	–0.055	0.026	–0.059	0.449	0.551	0.221	0.779
Croatia	0.133	–0.133	0.108	–0.010	0.647	0.353	0.964	0.036
Cyprus	0.136	–0.141	0.087	–0.015	0.757	0.243	0.935	0.065
Czechia	0.380	–0.470	0.200	–0.156	0.612	0.388	0.669	0.331
Denmark	0.304	–0.432	0.115	–0.244	0.442	0.558	0.313	0.687
Estonia	0.097	–0.237	0.028	–0.129	0.192	0.808	0.098	0.902
Finland	0.324	–0.592	0.234	–0.041	0.684	0.316	0.960	0.040
France	0.292	–0.075	0.353		0.836	0.164	1	0
Germany	0.207	–0.156	0.167		0.689	0.311	1	0
Greece	0.218	–0.111	0.225		0.754	0.246	1	0
Hungary	0.367	–0.416	0.521		0.812	0.188	1	0
Iceland*	0.638	–0.316	0.594	–0.038	0.732	0.268	0.987	0.013
Ireland*	0.364	–0.364	0.383		0.800	0.200	1	0
Italy*	0.147	–0.018	0.218		0.927	0.073	1	0
Latvia	0.076	–0.189		–0.117	0.234	0.766	0	1
Lithuania	0.107	–0.084	0.079	–0.020	0.747	0.253	0.749	0.251
Luxembourg	0.276	–0.047	0.343		0.950	0.050	1	0
Malta	0.106	–0.061	0.045	–0.034	0.582	0.418	0.446	0.554
Netherlands	0.293	–0.364	0.238	–0.262	0.482	0.518	0.481	0.519
Norway	0.359	–0.299	0.249	–0.161	0.547	0.453	0.586	0.414
Poland	0.202	–0.192	0.198	–0.010	0.710	0.290	0.987	0.013
Portugal	0.148	–0.118	0.028	–0.045	0.464	0.536	0.240	0.760
Romania	0.070	–0.022	0.061	–0.023	0.851	0.149	0.874	0.126
Serbia	0.066	–0.050	0.040	–0.060	0.614	0.386	0.497	0.503
Slovakia	0.395	–0.091	0.466		0.840	0.160	1	0
Slovenia	0.295	–0.384	0.212	–0.085	0.609	0.391	0.781	0.219
Spain	0.110	–0.024	0.103		0.926	0.074	1	0
Sweden	0.185	–0.128	0.108	–0.115	0.587	0.413	0.447	0.553
Switzerland	0.293	–0.088	0.274		0.832	0.168	1	0
United Kingdom*	0.236	–0.086	0.297		0.930	0.070	1	0

Notes: The table reports the measure of relative sensitivity of the poverty headcount ratio to the child weight distinctly estimated for positive (β^+) and negative (β^-) changes and the proportions of marginal positive (β^+) and negative (β^-) changes on which the associated measures of relative sensitivity are estimated. Columns 1–2 and 5–6 are based on the whole range of weights, i.e., the interval [0; 1]. Columns 3–4 and 7–8 are based on a limited range of weights, i.e., the interval [0.3; 0.7].

* indicates that 2019 data was not available, and 2018 results are reported

Source: EU-SILC 2018-2019 (Cross UDB, 2020-09 version).

4.3 Robustness of headcount income poverty rate comparisons over time

The main results based on point estimates reported in Table 5 (columns (2) and (4)) are calculated on N=100 non-zero weights as defined by Equation (2), corresponding to a precision of 0.01. Within the full range of weights (column (2)), the proportion of robust comparisons of income poverty headcount ratio (H) between 2018 and 2019 ranges between 0.503 (Estonia) and 1 (Serbia, Switzerland and the UK). 14 of 32 countries report a level of robustness above 0.9, indicating that the comparisons of income poverty rates are robust to at least 90% of all combinations of adult-child weights. As noted, considering the whole range of weights between 0 and 1 may not necessarily be reasonable in practical applications. When considering a limited range of weights from the interval [0.3; 0.7], full robustness is achieved in 16 countries; while the measure of robustness exceeds 0.9 in five additional countries (column (4)).

In addition to results based on point estimates, we also report results including interval estimates of income poverty levels (columns (3) and (5)). Considering interval estimates in the whole range of weights results in substantially lower levels of robustness, with the highest value reported in the UK (0.81), and zero robustness for 13 countries. When the limited set of weights (column (5)) is considered (between 0.3 and 0.7), full robustness is reported in Croatia and the UK, whereas the number of countries with zero robustness increases to 21. This finding suggests that when standard errors of headcount income poverty rate estimates are considered in calculations, the robustness of comparisons drops considerably. Ultimately, the observed change in point estimates of the income poverty rate may mean that point estimates are not fully reliable.

In supplementary materials (Table S.A) we further report the same set of results as in Table 5 over five years, enabling comparisons across time for all countries in the study.

Table 5. Robustness of comparisons of changes in *H* between 2018 and 2019

Country	H_{2019}-H_{2018} [%]	Whole range		Limited range	
		Point	Interval	Point	Interval
	(1)	(2)	(3)	(4)	(5)
Austria	–0.963	0.776	0.000	0.944	0.000
Belgium	–1.622	0.989	0.342	1.000	0.250
Bulgaria	0.674	0.739	0.018	0.762	0.000
Croatia	–1.010	0.996	0.717	1.000	1.000
Cyprus	–0.645	0.730	0.136	0.866	0.000
Czechia	0.555	0.916	0.007	1.000	0.000
Denmark	–0.211	0.771	0.000	0.871	0.000
Estonia	–0.235	0.503	0.000	0.779	0.000
Finland	–0.376	0.720	0.000	0.673	0.000
France	0.242	0.698	0.041	0.843	0.000
Germany	–0.858	0.997	0.000	1.000	0.000
Greece	–0.677	0.917	0.299	1.000	0.766
Hungary	–0.454	0.630	0.048	0.800	0.000
Iceland*	–1.249	0.923	0.000	1.000	0.000
Ireland*	–0.724	0.769	0.134	0.911	0.016
Italy*	0.018	0.732	0.032	0.777	0.000
Latvia	–0.446	0.894	0.029	1.000	0.000
Lithuania	–2.336	0.794	0.236	1.000	0.172
Luxembourg	0.835	0.867	0.000	0.983	0.000
Malta	0.428	0.920	0.000	1.000	0.000
Netherlands	–0.056	0.733	0.000	0.942	0.000
Norway	–0.209	0.822	0.000	1.000	0.000
Poland	0.599	0.935	0.091	1.000	0.344
Portugal	–0.024	0.726	0.000	0.636	0.000
Romania	0.294	0.749	0.034	0.729	0.000
Serbia	–1.094	1.000	0.327	1.000	0.438
Slovakia	–0.353	0.922	0.238	0.770	0.016
Slovenia	–1.379	0.969	0.435	1.000	0.984
Spain	–0.886	0.799	0.000	0.961	0.000
Sweden	0.676	0.934	0.000	1.000	0.000
Switzerland	1.448	1.000	0.315	1.000	0.172
Uited Kingdom*	1.522	1.000	0.810	1.000	1.000

Notes: Results are based on the precision of 0.01, referring to N=100 in Equation (2). *Whole range* refers to the whole set of weights [0; 1]; *Limited range* refers to weights in [0.3; 0.7]. *Point* [*Interval*] refers to *point* [*interval*] estimates of *H*.

* indicates that 2019 data was not available, and 2017 vs 2018 comparisons are reported

Source: Author's calculations based on EU-SILC 2018-2019 data (Cross UDB, 2020-09 version).

V. CONCLUDING REMARKS

This paper complements existing literature on the adoption of equivalence scales in income poverty analyses. We first touch on the sensitivity of income poverty indicators to parameters of equivalence scales and then elaborate on the robustness of income poverty rates comparisons. We propose two practical measures. The first, the measure of the sensitivity of an income poverty indicator to an equivalence scale, is constructed as an average change in an income poverty indicator associated with a change in the parameters of an equivalence scale (weights assigned to second and next adults and children in a household). The second measure, of robustness of comparisons in time or space assesses the share of combinations leading to the same outcome (an increase or a decrease in the indicator value) when scale parameters are changed. It is based on combinations of the equivalence scale parameters (adult/child weights) that lead to the same conclusion: an increase or a decrease in the indicator value.

We demonstrate our approach using the 2015-2019 EU-SILC data, exploring the sensitivity of the income poverty headcount ratio, also referred to as the at-risk-of-poverty rate, to an OECD-type equivalence scale. In addition to examining sensitivity, we assess the robustness of comparisons of income poverty headcount ratios over time. Traditionally, this type of equivalence scale assigns a value of 1 to the first adult household member and then it distinguishes between weights assigned to other adult and child household members.

Previous research has raised concerns about arbitrarily chosen weights, and there is also an ongoing debate regarding the sensitivity of income poverty and inequality indicators to alternative equivalence scales. While an assessment of the sensitivity and robustness of income poverty indicators (comparisons) to equivalence scales in existing studies is usually based on a limited set of equivalence scales, our approach investigates a wide range of scales. Using 2019 data, we show that the relative sensitivity of the income poverty rate to the weight of second and other adults in a household ranges between 0.145 (Spain and Serbia) and 0.855 (Czechia), while the relative sensitivity to the weight of children ranges between 0.058 (Serbia) and 0.557 (Iceland). The results suggest that a 1-percentage-point increase in the weight of an adult in the household is associated with at least 0.145 percentage change in the value of the income poverty rate (in Spain and Serbia), while a 1-percentage-point increase in the child weight causes at least 0.058 percentage change in the income poverty rate (in Serbia). We further show that 14 of 32 countries report year-on-year comparisons of income poverty rates which are robust to at least 90% of all combinations of adult-child weights. However, when interval estimates are used instead of point estimates (i.e., standard errors of income poverty estimates are employed in the calculations), the robustness of comparisons drops considerably.

High sensitivity of an income poverty indicator and low robustness of the income poverty indicator comparisons to parameters of an equivalence scale ultimately calls into question its appropriateness to inform social policy. This may be a particular concern when equivalence scales are arbitrarily chosen, as is the case of income poverty measurement in the European Union. If two values of an income poverty indicator based on two slightly different sets of weights differ substantially, the legitimacy of either of the two values can be questionable. Understanding the sensitivity of the income poverty indicator and the robustness of comparisons to equivalence scales is an essential pre-assumption for drawing conclusions relating to the values of income poverty indices. The more sensitive an income poverty indicator is to equivalence scales, the more carefully interpretations of its value need to be communicated. Similarly, comparisons of income poverty levels over time which are not robust to different (though reasonable) sets of weights assigned to adult/child household members raise concerns in situations such as tracking poverty alleviation and assessment of anti-poverty policies.

We demonstrate our approach using the income poverty headcount ratio as the simplest and most widely used indicator of poverty. Nevertheless, the simplicity of the measures proposed in this paper allows their practical application to a wide range of income poverty and inequality indicators. The approach proposed in this study can be applied not only to assess the sensitivity of income poverty indices (see, e.g., Zheng, 2002), but also to income inequality measures (e.g., Jenkins and Van Kerm, 2009).

The paper contributes to the literature examining the robustness of poverty comparisons. Full robustness assessment requires consideration of numerous attributes which are often arbitrarily chosen, such as poverty threshold, equivalence scale, and the age definition of an adult household member. Clearly, the more attributes are considered, the more likely it is that full robustness will be ruled out. Hence, when it comes to practical applications of a sensitivity assessment, a limited, reasonable set of parameters should be considered. Such an approach is demonstrated in this paper, where, in addition to the results based on the full range of weights, we also report results based on a range of weights between 0.3 and 0.7 for both adults and children. These results are again mixed – while data from some countries experience lower levels of sensitivity to the limited range of weights, other countries experience the opposite. However, considering a limited range of weights suggests higher levels of robustness of comparisons of income poverty rates over time, which is again an important finding from the perspective of policy implications.

We are aware of problematic facets of equivalence scales and arbitrariness in setting the parameters, but in this paper, our aim is not to propose an "optimal" equivalence scale, but rather to examine the sensitivity and robustness of income poverty indicators (comparisons) to one of the most widely used equivalence scales – the OECD-type equivalence scale.

This paper also considers assessment of the sensitivity of income poverty indicators to equivalence scales from a perspective of different parameters. The age at which a household member is counted as an adult is one such parameter. We outlined an assessment of sensitivity of comparisons to the definition of an adult household member, and the data suggests high levels of correlation between results based on the threshold of 14 years of age and alternative thresholds of 15, 16, 17, and 18. Gender is another example of such a parameter, which could be considered in future studies. As noted by Ravallion (2016), hardly any equivalence scale in practical use considers different parameters for males and females. Clearly, different parameters can be derived for subpopulations such as gender and age groups, and for subpopulations based, for example, on economic activity status. In any effort to derive such equivalence scales, one needs to consider the balance between increased precision in estimations of income poverty levels and the practical applicability of enhanced estimations.

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